Ann. For. Res. 66(1): 4617-4634, 2023
ISSN: 18448135, 20652445

ANNALS OF FOREST RESEARCH
www.e-afr.org

AN ANALYSIS OF THE SURVIVAL FUNCTION FOR XGD OF PATIENTS WITH COVID-19 USING BAYESIAN AND MAXIMUM LIKELIHOOD METHODS

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Abstract

This paper suggests a new probability distribution, called xgamma1 distribution. This distribution is generating as a mixture of exponential and gamma distributions which has one positive parameter (θ > 0). We studying and deriving the Mathematical, structural, and survival functions properties of the xgamma1 distribution and we use the Standard Bayesian and maximum likelihood estimation methods to estimate the new model parameter, and then it was compared the properties of Xgamma1 distribution with some other distributions such as the xgamma distribution and one parameter Lindley distribution. In addition we use the simulation for generating random samples and is found that the Standard Bayesian method is the best in estimating the survival function for the new distribution. Finally we found that Xgamma1 provides a better fit to data than xgamma and Lindley models when analyzing patients infected with the Covid-19 virus in AL-Najaf al-Ashraf, because achieved the lowest criterions (AIC, AICc, BIC) than other distributions.

Keywords: Xgamma distribution, Xgamma1 distribution, Survival function, Standard Bayes estimation, Maximum Likelihood estimation.

INTRODUCTION

Medicine, biology, public health, epidemiology, engineering, economics, and demography are just a few of the fields where time-to-event data must be analyzed. Though all these disciplines can benefit from the statistical tools we present, we will focus on applying them to biology and medicine.

There have been several new models developed in recent years to provide richness and a degree of accuracy that allows them to be fitted to complex datasets. The real-life phenomenon was modeled using new statistical models that incorporated a mixture of probability distributions. Finite mixture densities have also been widely used to model a variety of data sets. As a result, Researcher Subhradev Sen et al. in 2016 proposed a new, one-parameter probability distribution a combination of exponential and gamma distributions with specified weights, called the Xgamma Distribution (XGD). Then he derived many mathematical and structural properties such as moments, measures of skewness and kurtosis, after which important survival characteristics such as risk rate and mean Residual life. Follow this by estimating the distribution parameter in two ways: MLE and MOM. Then he ran a Monte Carlo simulation to generate a random sample with an XGD distribution and estimate the parameter using the mentioned methods. Finally, compare the new distribution to the exponential distribution by analyzing a real-world dataset on patient rest times (hours) for 20 patients receiving analgesics. The Xgamma distribution found to provide a better fit to the data than the exponential distribution[1]. In (2018) researcher (Maiti, S. S.) et al. derived two discontinuous forms of the Xgamma distribution, Xgamma-I and Xgamma-II using two different methods, Discrete Concentration Approach and Discrete Analogue Approach. The mathematical properties and survival properties of these distributions were studied. The parameter was estimated by the methods of MOM and MLE. The discrete Xgamma-I and Xgamma-II distributions were compared with the Poisson distribution, the negative binomial distribution, and the complex discontinuous Lindley distribution using six data sets[2]. In the year (2018) Subhradev Sen studied additional properties (eg, characteristic function and generation function) and some other survival properties (eg mean time to failure). The unknown parameter was estimated using the maximum potential method and the Bayes method, assuming the previous gamma distribution under the Censoring system. A simulation study was conducted to compare the estimates described in the two estimation processes. Conclude that use Bayes' estimate if better advance information can be obtained; Otherwise, MLE would be a better choice. Real data representing survival times (in days) of 72 guinea pigs infected with Bacillus tuberculosis were also analyzed to compare the Xgamma model with different age models and Xgamma performance was found to be quite satisfactory[3]. In the year (2021) (Saha) and others proposed four classical methods for estimating the parameter of the XGD distribution, which are (MLE method, Ordinary least squares OLSE, Weighted least squares WLSE, Maximum divergence product MPS) and Bayesian estimation method with gamma distribution as previous distribution and general entropy function as a loss function. A simulation study was conducted to compare these methods with different sample sizes and different combinations of unknown parameters. It was observed that the Bayes estimation of survival characteristics is better compared to the classical estimation processes. The data set of rest times (in hours) of 20 patients receiving analgesics was analyzed for the purpose of clarification[4]. This is followed by the xgamma1 distribution being offered by us.

For lifetime studies, estimation of the survival function and the hazard function for the probability distributions used for modeling lifetime data are important tools. S(t) represents the complement of the distribution function F(t), which is the survival function. [5]

1. Xgamma distribution:

Sen et al., 2016 proposed a special mixture of exponential (θ) and gamma (3,θ) distributions, denoted XGD, an assuming that the two parameters of mixing are $\pi_1 = \frac{\theta}{1+\theta}, \quad \pi_2 = 1-\pi_1 = \frac{1}{1+\theta}$. He derived the survival function, as well as inferences of the XGD [6].

T is a random variable then the XGD is define as follow:

$f\,(\mathrm{t}) = \pi_1 f_1(t;\theta) + \pi_2 f_2(t;\theta)$

$$f\,(\mathrm{t}) = \frac{\theta}{1+\theta}\left(\theta\, e^{-\theta t}\right) + \frac{1}{1+\theta}\left(\frac{\theta^3 t^2}{\rceil 3} e^{-\theta t}\right)$$

$$f\,(\mathrm{t};\theta) = \frac{\theta^2}{(1+\theta)}\left(1+\frac{\theta}{2}t^2\right)e^{-\theta t} \quad ,t>0,\theta>0$$

(1)

This is represented by T ~ xgamma (θ).
The cumulative density function (CDF) of T is given by

$$F\,(\mathrm{t}) = 1 - \frac{\left(1+\theta+\theta\, t+\frac{\theta^2 t^2}{2}\right)}{(1+\theta)}e^{-\theta t} \quad ,t>0,\theta>0$$

(2)

The survival function S(t) and the hazard function h(t) are defined a follow

$$S\,(t) = \frac{\left(1+\theta+\theta\, t+\frac{\theta^2 t^2}{2}\right)}{(1+\theta)}e^{-\theta t} \quad ,t>0,\theta>0$$

(3)

$$h\,(t) = \frac{\left(1+\frac{\theta\, t^2}{2}\right)\theta^2}{(1+\theta+\theta\, t+\frac{\theta^2 t^2}{2})}$$

(4)

The moments and the variance and the other properties are defined a follow:

(5)

$$\mu_1' = \frac{(\theta+3)}{\theta(1+\theta)}$$

(6)

$$\mathrm{var(t)} = \frac{\theta^2+8\theta+3}{\theta^2(1+\theta)^2}$$

(7)

$$Mode = \begin{cases} \frac{1+\sqrt{1-2\theta}}{\theta}, & 0<\theta\le 0.5 \\ 0, & otherwise \end{cases}$$

(8)

$$\Upsilon_1 = \frac{2(\theta^3 + 15\theta^3 + 9\theta + 3)}{\left(\theta^2 + 8\theta + 3\right)^{3/2}}$$

(9)

$$\Upsilon_2 = \frac{3(3\theta^4 + 64\theta^3 + 102\theta^2 + 72\theta + 15)}{\left(\theta^2 + 8\theta + 3\right)^2}$$

(10)

2. Xgamma1 distribution (XG1):

In this section we suggest a new formula for Xgamma distribution, this is the new formula proposed by us for the Xgamma distribution, we refer to it as the Xgamma1 distribution, and denoted as XG1D. Suppose that we have gamma distribution with parameter (3, λ=1/θ) and exponential distribution with parameter (λ=1/θ) and let the mixing equation is:

$$f(t;\theta) = \pi_1 f_1(t) + \pi_2 f_2(t)$$

Where

$$\pi_1 = \frac{\theta}{1+\theta}, \quad and \quad \pi_2 = 1 - \pi_1 = \frac{1}{1+\theta}$$

$$f_1(t) = \frac{1}{\theta} e^{-\frac{t}{\theta}} \quad and \quad f_2(\text{t}) = \frac{t^2}{\theta^3 \Gamma 3} e^{-\frac{t}{\theta}}$$ then:

$$f(t;\theta) = \frac{\theta}{1+\theta}\left(\frac{1}{\theta} e^{-\frac{t}{\theta}}\right) + \frac{1}{1+\theta}\left(\frac{t^2}{\theta^3 \Gamma 3} e^{-\frac{t}{\theta}}\right)$$

$$f(t;\theta) = \frac{1}{1+\theta} e^{-\frac{t}{\theta}} \left(1 + \frac{t^2}{2\theta^3}\right), \quad t, \theta > 0$$

(11)

This is represented by T ~ xgamma1 (θ), and is denoted a XG1D.

The corresponding cumulative distribution function (C.D.F) of the XG1 is:

$$F(t;\theta) = 1 - e^{-\frac{t}{\theta}} \left(1 + \frac{(2\theta + t)t}{2\theta^2(1+\theta)}\right), \quad t, \theta > 0$$

(12)

The survival S(t) function and hazard function h(t) are given by

$$S(t) = \left(1 + \frac{t(2\theta + \text{t})}{2\theta^2(1+\theta)}\right) e^{-\frac{t}{\theta}} \quad , t > 0, \theta > 0$$

(13)

$$h(t)=\frac{2\theta^3+t^2}{2\theta^3(1+\theta)+\theta t(2\theta+t)} \tag{14}$$

The rth moments of x about zero is:

$$\mu_r'=\frac{r!\theta^r}{(1+\theta)}\left(\theta+\frac{(r+2)(r+1)}{2}\right)\quad for\quad r=1,2,... \tag{15}$$

The mean and the variance and the other properties are defined a follow:

$$\mu_1'=\frac{\theta(3+\theta)}{(1+\theta)} \tag{16}$$

$$\text{var}(x)=\frac{\theta^2(\theta^2+8\theta+3)}{(1+\theta)^2} \tag{17}$$

$$Mode=\begin{cases}-\theta+\theta\sqrt{1+2\theta}, & 0<\theta<\infty\\ 0, & otherwise\end{cases} \tag{18}$$

$$\Upsilon_1=\frac{\dfrac{2\theta^3(\theta^3+15\theta^3+9\theta+3)}{(1+\theta)^3}}{\dfrac{\theta^3\left(\theta^2+8\theta+3\right)^{2/3}}{(1+\theta)^3}} \tag{19}$$

$$\Upsilon_2=\frac{3(5\theta^4+94\theta^3+216\theta^2+218\theta+75)}{\left(\theta^2+8\theta+3\right)^2} \tag{20}$$

$$\therefore M_x(t)=\frac{1}{(1+\theta)}\left(\frac{1}{(\frac{1}{\theta}-t)}+\frac{1}{\theta^3\left(\frac{1}{\theta}-t\right)^3}\right);t\in R \tag{21}$$

3. The Lindley Distribution (LD):

Lindley invented a single-parameter distribution as a continuous probability distribution consisting of mixing the gamma (2,θ) distribution and the Exponential (θ) distribution, which is described by its probability density function [7] [8]:

$$f(x;\theta)=\frac{\theta^2}{(1+\theta)}(1+x)e^{-\theta x} \quad ,x>0,\theta>0 \tag{22}$$

This is represented by X ~ LD (θ).
The cumulative density function (CDF) of X is given by

$$F(x)=1-\frac{(1+\theta+\theta x)}{(1+\theta)}e^{-\theta x} \quad ,x>0,\theta>0 \tag{23}$$

The survival function S(t) and the hazard function h(t) are given by

$$S(t)=\frac{(1+\theta+\theta t+)}{(1+\theta)}e^{-\theta t} \quad ,t>0,\theta>0 \tag{24}$$

$$h(t)=\frac{(1+t)\theta^2}{(1+\theta+\theta t)} \tag{25}$$

The moments and the variance and the other properties are defined a follow:

$$\mu'_r=\frac{r!(\theta+\mathrm{r}+1)}{\theta^r(1+\theta)} \quad for \quad r=1,2,3,... \tag{26}$$

$$\mu'_1=\frac{(\theta+2)}{\theta(1+\theta)} \tag{27}$$

$$\mathrm{var(t)}=\frac{\theta^2+4\theta+2}{\theta^2(1+\theta)^2} \tag{28}$$

$$\mathrm{mod}e=\frac{1-\theta}{\theta}, \quad \theta<1 \tag{29}$$

$$\Upsilon_1=\frac{2(\theta^3+6\theta^3+6\theta+2)}{(\theta^2+4\theta+2)^{3/2}} \tag{30}$$

$$\Upsilon_2=\frac{3(3\theta^4+24\theta^3+44\theta^3+32\theta+8)}{\left(\theta^2+4\theta+2\right)^2} \tag{31}$$

4. Estimation of the Parameter

4.1 The maximum Likelihood Estimation:

The maximum likelihood method is widely used in statistical estimation. There are differing opinions about who proposed the method first. While Fisher invented the name of maximum likelihood, who spread the use of it widely, and demonstrated its optimality properties.[9]
We calculate it for the xgamma1 distribution as follows:
Assuming that (T1, T2, ..., Tn) is a random sample of size n from the Xgamma1 distribution, the estimate for the parameter θ can be found by the MLE as follows:

$$L=\prod_{i=1}^{n} f\,(t_i;\theta)$$

$$L=\frac{1}{(1+\theta)^n}\prod_{i=1}^{n}\left(1+\frac{t_i^2}{2\theta^3}\right)e^{-\frac{t_i}{\theta}}$$

$$\text{lnL}=\sum_{i=1}^{n}\ln\left(1+\frac{t_i^2}{2\theta^3}\right)-n\ln(1+\theta)-\frac{1}{\theta}\sum_{i=1}^{n}t_i$$

Let $l(\theta;t)=\frac{\partial\,\text{lnL}}{\partial\theta}=0$

$$\sum_{i=1}^{n}\frac{-3t_i^2}{\theta\left(2\theta^3+t_i^2\right)}-\frac{n}{1+\theta}+\frac{1}{\theta^2}\sum_{i=1}^{n}t_i=0 \tag{32}$$

Since (32) is a non-linear equation, it cannot be solved analytically; hence, numerical methods are used, such as the Newton-Raphson algorithm .
Let the initial solution is:

$$\theta_0=\frac{n}{\sum_{i=1}^{n}t_i}$$

$$\theta^{(i)}=\theta^{(i-1)}-\frac{l(\theta|x)}{l'(\theta|x)}$$

When $\theta(i)\cong\theta(i-1)$, we choose $\theta_{mle}=\theta(i)$. Then the MLE for the survival function and the hazard rate function defined as follows:

$$\hat{S}(t)=\left(1+\frac{t(2\hat{\theta}_{mle}+t)}{2\hat{\theta}^{2}_{mle}(1+\hat{\theta}_{mle})}\right)e^{-\frac{t}{\hat{\theta}_{mle}}}$$

(33)

$$\hat{h}(t)=\frac{\hat{\theta}_{MLE}(1+\frac{t^{2}\hat{\theta}^{3}_{MLE}}{2})}{\left(1+\hat{\theta}_{MLE}+t\hat{\theta}^{2}_{MLE}+\frac{t^{2}\hat{\theta}^{3}_{MLE}}{2}\right)}$$

(34)

4.2 Standard Bayesian Estimation method:

In this section we shall use the standard Bayesian estimation method to estimate the parameter θ of the XG1D.

Suppose that the prior distribution of the parameter θ is gamma distribution defined as follows:

$$p(\theta)=\frac{\theta^{\alpha-1}}{\beta^{\alpha}\Gamma\alpha}e^{-\frac{\theta}{\beta}};\quad \alpha,\beta>0$$

And let a squared error loss function, then we can compute the $\hat{\theta}_{Bayes}$ as follows:

$$f(\theta|t_1,t_2,...,t_n)=\frac{L(t_1,t_2,...,t_n|\theta)\mathrm{P}(\theta)}{\int_0^{\infty}L(t_1,t_2,...,t_n|\theta)\mathrm{P}(\theta)d\theta}$$

$$\mathrm{f}(\theta|t_1,t_2,...,t_n)=\frac{\frac{\theta^{\alpha-1}}{(1+\theta)^{n}}e^{-\frac{1}{\theta}\left(\frac{\theta^{2}}{\beta}+\sum_{i=1}^{n}t_i\right)}\prod_{i=1}^{n}\left(1+\frac{t_i^{2}}{2\theta^{3}}\right)}{\int_0^{\infty}\frac{\theta^{\alpha-1}}{(1+\theta)^{n}}e^{-\frac{1}{\theta}\left(\frac{\theta^{2}}{\beta}+\sum_{i=1}^{n}t_i\right)}\prod_{i=1}^{n}\left(1+\frac{t_i^{2}}{2\theta^{3}}\right)d\theta}$$

(35)

$$Risk=E(\hat{\theta}-\theta)^{2}$$

$$\frac{\partial Risk}{\partial\theta}=0$$

$$\hat{\theta}_{Bayes}=E(\theta|T)$$

$$\breve{\theta}_{Bayes}=\frac{\frac{\theta^{\alpha}}{(1+\theta)^{n}}e^{-\frac{1}{\theta}\left(\frac{\theta^{2}}{\beta}+\sum_{i=1}^{n}t_{i}\right)}\prod_{i=1}^{n}\left(1+\frac{t_{i}^{2}}{2\theta^{3}}\right)}{\int_{0}^{\infty}\frac{\theta^{\alpha-1}}{(1+\theta)^{n}}e^{-\frac{1}{\theta}\left(\frac{\theta^{2}}{\beta}+\sum_{i=1}^{n}t_{i}\right)}\prod_{i=1}^{n}\left(1+\frac{t_{i}^{2}}{2\theta^{3}}\right)d\theta} \quad (36)$$

Since the equation (36) is not closed formula, so we cannot compute it numerically and we will using Lindley's approximation method to calculate the integral to find the Bayes estimate for the parameter θ of XG1D.

And so we can compute the Bayes estimation for the survival function and the hazard rate function for XG1D as follows:

$$\breve{S}(t)=\left(1+\frac{t(2\breve{\theta}_{Bayes}+\mathrm{t})}{2\breve{\theta}_{Bayes}^{2}(1+\breve{\theta}_{Bayes})}\right)e^{-\frac{t}{\breve{\theta}_{Bayes}}} \quad (37)$$

$$\breve{h}(t)=\frac{\hat{\theta}_{per}(1+\frac{t^{2}\hat{\theta}_{per}^{3}}{2})}{\left(1+\hat{\theta}_{per}+t\hat{\theta}_{per}^{2}+\frac{t^{2}\hat{\theta}_{per}^{3}}{2}\right)} \quad (38)$$

5. Simulation study

For examining the behavior of selected estimators and survival functions of the xgamma1 distribution, a Monte Carlo simulation was conducted with M = 1000 iterations. A sample size of 10, 50, and 100 was considered, and values of (θ) were taken as 0.5, 1, and 5, and we use the IMSE measure to select the appropriate one Calculated measure include:

$$IMSE\left(\breve{S}(t)\right)=\frac{1}{M}\sum_{i=1}^{M}\left[\frac{1}{n_{i}}\sum_{j=1}^{n_{i}}\left(\breve{S}_{i}(t_{j})-S_{i}(t_{j})\right)^{2}\right] \quad (39)$$

5.1 generation random data

5.1.1 For generation of random data based on the xgamma, see [1].

5.1.2 For generation of random data T_i, i = 1, 2,..., n based on the xgamma1, use this algorithm:

1. Generate $A_i \sim uniform(0,\ 1), i = 1,\ 2,\ldots,n$

2. Generate $V_i \sim exponential\left(\lambda = \frac{1}{\theta}\right), i = 1, 2, \ldots, n$

3. Generate $W_i \sim gamma\left(3, \lambda = \frac{1}{\theta}\right), i = 1, 2, \ldots, n$

4. Generate $T_i \sim Xgamma1\left(\lambda = \frac{1}{\theta}\right), i = 1, 2, \ldots, n$ by the following:

If $A_i \le \frac{\theta}{(1+\theta)}$, then set $T_i = V_i$, Otherwise, set $T_i = W_i$.

5.1.3 For a generation of random data based on the Lindley distribution, see[8]

Table1. Simulation results when α=1, β=1

n	θ	Dis.	XG		XG1		LD	
			mle	bayes	mle	bayes	mle	bayes
10	0.5	$\hat{\theta}$	0.5219755	0.5317348	0.4965105	0.5196097	0.5282134	0.5417303
		S_{real}	0.4971737		0.5063518		0.4980126	
		$\hat{S}$	0.4895564	0.4815998	0.4888594	0.5011094	0.4901369	0.4802006
		IMSE	0.0074392	0.00735954	0.00653241	**0.00531164**	0.00869610	0.00854808
	1	$\hat{\theta}$	1.057734	1.052141	0.9670008	1.010342	1.076775	1.07136
		S_{real}	0.499371		0.5006836		0.5042522	
		$\hat{S}$	0.4898243	0.4905981	0.482928	0.4864883	0.4906965	0.4908986
		IMSE	0.0078871	0.00707214	0.00514875	**0.00451177**	0.00841974	0.00740072
	5	$\hat{\theta}$	5.55726	3.427065	4.851119	3.944204	5.567088	3.30691
		S_{real}	0.5055966		0.5038691		0.5051448	
		$\hat{S}$	0.4875663	0.6164013	0.4832445	0.4519589	0.4890065	0.6211021
		IMSE	0.0078425	0.01638372	0.00845313	**0.00727652**	0.00829058	0.05691159
50	0.5	$\hat{\theta}$	0.5052668	0.507299	0.5010488	0.5056749	0.505661	0.5084954
		S_{real}	0.4994295		0.4996453		0.5002987	
		$\hat{S}$	0.4970178	0.4953781	0.497599	0.4998245	0.4982902	0.4962469

		IMSE	0.00157432	0.001575222	0.001141831	**0.001102606**	0.001641886	0.001640769
	1	$\hat{\theta}$	1.009406	1.009422	0.9947888	0.9976153	1.016688	1.017168
		S_{real}	0.4997515		0.5014555		0.5010455	
		$\hat{S}$	0.4982329	0.4981918	0.4980354	0.4985911	0.4976884	0.4974728
		IMSE	0.00148354	0.001456606	0.000933738	**0.000910859**	0.001709498	0.001673523
	5	$\hat{\theta}$	5.061953	4.758785	5.03028	4.873884	5.068565	4.751043
		S_{real}	0.4980705		0.4991366		0.5000012	
		$\hat{S}$	0.4970876	0.5147685	0.4977429	0.492093	0.4985371	0.5168229
		IMSE	0.00151286	0.001585417	0.001343485	**0.001254242**	0.00149018	0.001564172
100	0.5	$\hat{\theta}$	0.5014599	0.5024804	0.4989553	0.5012633	0.5017472	0.5031692
		S_{real}	0.499229		0.4988413		0.4997643	
		$\hat{S}$	0.4989253	0.4981025	0.4972541	0.4983615	0.4994771	0.4984506
		IMSE	0.00074697	0.000745609	0.000489968	**0.000479802**	0.000797462	0.000795705
	1	$\hat{\theta}$	1.005742	1.005794	0.9953444	0.9967788	1.008064	1.008395
		S_{real}	0.4999859		0.5004381		0.5004905	
		$\hat{S}$	0.498878	0.4988501	0.4984252	0.4986953	0.4987465	0.498622
		IMSE	0.00076193	0.000755019	0.000442662	**0.000437031**	0.000770973	0.000763088
	5	$\hat{\theta}$	5.052831	4.904046	5.017813	4.942618	5.003853	4.851813
		S_{real}	0.5006508		0.4992884		0.4975397	
		$\hat{S}$	0.4987386	0.5073576	0.4986785	0.4959197	0.4985814	0.5074085
		IMSE	0.00072809	0.000717032	0.000659297	**0.000637299**	0.000780793	0.000828847

Table2. Simulation results when α=1, β=0.5

n	θ	Dis.	XG		XG1		LD	
			mle	bayes	mle	bayes	mle	bayes
10	0.5	$\hat{\theta}$	0.5272115	0.5430306	0.5084563	0.5268579	0.5252584	0.5466954
		S_{real}	0.5006369		0.4860069		0.4967864	
		$\hat{S}$	0.4883279	0.4760874	0.479263	0.4893184	0.4895861	0.4746167
		IMSE	0.00832812	0.0087165 36	0.0055328 48	**0.0046584 92**	0.0082513 83	0.0085814 07
	1	$\hat{\theta}$	1.046745	1.069967	0.983931	0.9791125	1.067758	1.098754
		S_{real}	0.4948549		0.5004433		0.5001215	
		$\hat{S}$	0.4899485	0.4812888	0.4853532	0.4859935	0.4897408	0.4790714
		IMSE	0.00831512	0.0081362 95	0.0050356 31	**0.0043983 82**	0.0083709 81	0.0082961 12
	5	$\hat{\theta}$	5.601064	4.713552	5.875169	3.7075	5.405259	4.601945
		S_{real}	0.5054272		0.5050804		0.4970632	
		$\hat{S}$	0.4864003	0.5296482	0.5308254	0.4603386	0.4870257	0.5280108
		IMSE	0.00827919	0.0044595 13	0.0008642 66	**0.0027139 46**	0.0073149 59	0.0047922 49
50	0.5	$\hat{\theta}$	0.5052226	0.5083183	0.5038753	0.5078001	0.5041062	0.5083022
		S_{real}	0.4997982		0.4968554		0.4982381	
		$\hat{S}$	0.497318	0.4948431	0.4963427	0.4982371	0.4974948	0.4944749
		IMSE	0.0015164	0.0015338 2	0.0011056	**0.0010726 3**	0.0016909 9	0.0016997 5
	1	$\hat{\theta}$	1.009553	1.014472	0.9925037	0.9922295	1.013734	1.020227
		S_{real}	0.4998401		0.5019474		0.5004572	
		$\hat{S}$	0.4983678	0.4965666	0.4981933	0.4982031	0.4979926	0.4957702
		IMSE	0.0015336	0.0015314 2	0.0009219	**0.0008991 6**	0.0016029	0.0016056 8
	5	$\hat{\theta}$	5.081158	4.967714	4.956516	4.650964	5.085123	4.967541
		S_{real}	0.499807		0.5013577		0.5003047	
		$\hat{S}$	0.4975089	0.5039016	0.4971336	0.4858032	0.4979432	0.5044931

		IMSE	0.0014541	0.00135037	0.00139532	**0.00134610**	0.00153865	0.00142466
100	0.5	$\hat{\theta}$	0.5023293	0.5038719	0.4978518	0.4998514	0.5019401	0.5040334
		S_{real}	0.4994875		0.5019869		0.4989826	
		$\hat{S}$	0.4985592	0.49732	0.4997535	0.5007153	0.4986056	0.4970954
		IMSE	0.00079086	0.000794177	0.000539448	**0.000526518**	0.000830880	0.000833289
	1	$\hat{\theta}$	1.00608	1.00855	1.002249	1.002101	1.005584	1.008839
		S_{real}	0.500198		0.4998759		0.500124	
		$\hat{S}$	0.4989265	0.4980224	0.4990729	0.4990609	0.4992308	0.498114
		IMSE	0.00073657	0.000737354	0.000416031	**0.000410849**	0.000787415	0.000787415
	5	$\hat{\theta}$	5.033109	4.97879	4.998443	4.84651	5.043457	4.986964
		S_{real}	0.4993125		0.4996295		0.5002673	
		$\hat{S}$	0.4986577	0.5017678	0.4981267	0.4925113	0.4990974	0.5022887
		IMSE	0.00079577	0.000770610	0.000743288	**0.000721992**	0.000835798	0.000804714

Table3. Simulation results when α=0.5, β=1

n	θ	Dis.	XG		XG1		LD	
			mle	bayes	mle	bayes	mle	bayes
10	0.5	$\hat{\theta}$	0.5291089	0.5278206	0.5267029	0.5464428	0.5301062	0.529514
		S_{real}	0.5040521		0.5004259		0.500654	
		$\hat{S}$	0.4903893	0.4909493	0.4990677	0.5090659	0.4901951	0.4900011
		IMSE	0.00837347	0.008003467	0.006293766	**0.005342153**	0.008422155	0.007983399
	1	$\hat{\theta}$	1.068243	1.036167	0.9919376	0.9960593	1.076166	1.039069
		S_{real}	0.5031082		0.4994177		0.5038889	
		$\hat{S}$	0.4903319	0.5002973	0.4854722	0.4874329	0.4902669	0.5007657
		IMSE	0.00783091	0.006814802	0.005089852	**0.004522845**	0.008737225	0.007395025
	5	$\hat{\theta}$	5.460309	3.203952	4.91037	3.893084	5.444185	3.092649

		S_{real}	0.4992049		0.503365		0.4989155	
		$\hat{S}$	0.4864815	0.6306662	0.485695	0.450194	0.4885967	0.6374663
		IMSE	0.00773139	0.032489	0.008373097	**0.007239291**	0.008512138	0.02868551
50	0.5	$\hat{\theta}$	0.5075352	0.5074744	0.4959822	0.4999525	0.5051437	0.5052965
		S_{real}	0.501161		0.5006946		0.4995575	
		$\hat{S}$	0.4971233	0.4971562	0.4964103	0.4983609	0.4981287	0.4979968
		IMSE	0.00169403	0.001680328	0.001097764	**0.001049593**	0.001717629	0.001701009
	1	$\hat{\theta}$	1.014794	1.009913	1.000278	1.001531	1.01806	1.012682
		S_{real}	0.5016403		0.4996512		0.5020029	
		$\hat{S}$	0.4983258	0.5000556	0.4974356	0.4977023	0.4981368	0.4999065
		IMSE	0.00156305	0.001522571	0.000840429	**0.000822819**	0.001698877	0.00164713
	5	$\hat{\theta}$	5.06548	4.724565	4.993793	4.82512	5.089651	4.728643
		S_{real}	0.4983325		0.499563		0.4996084	
		$\hat{S}$	0.4970118	0.5170193	0.4967364	0.4905993	0.4974146	0.5182432
		IMSE	0.00155444	0.00169146	0.001428209	**0.001355391**	0.001700339	0.001804121
100	0.5	$\hat{\theta}$	0.5031848	0.5031688	0.4998147	0.5018153	0.5038235	0.503912
		S_{real}	0.5010716		0.5011191		0.5001199	
		$\hat{S}$	0.4994852	0.4994943	0.4998242	0.5007784	0.4982887	0.4982204
		IMSE	0.0008320	0.00082872	0.00053102	**0.00052123**	0.00077393	0.00077045
	1	$\hat{\theta}$	1.002556	1.000238	0.9984368	0.9990976	1.008235	1.005676
		S_{real}	0.4993118		0.4999459		0.4997247	
		$\hat{S}$	0.4995076	0.5003467	0.4985352	0.4986647	0.498034	0.4988983
		IMSE	0.0008147	0.00080816	0.00042057	**0.00041607**	0.00083319	0.00082120
	5	$\hat{\theta}$	5.03596	4.870007	4.995693	4.913916	5.030193	4.857001
		S_{real}	0.4996696		0.4992345		0.4991291	

		$\hat{S}$	0.4987883	0.5084604	0.4979517	0.4949249	0.498608	0.5086368
		IMSE	0.0007369	0.00076563	0.00063797	**0.00062223**	0.00074907	0.00079185

Table4. Simulation results when α=0.5, β=0.5

n	θ	Dis.	XG		XG1		LD	
			mle	bayes	mle	bayes	mle	bayes
10	0.5	$\hat{\theta}$	0.5175718	0.5224654	0.4843005	0.5008663	0.5304375	0.5378162
		S_{real}	0.4957262		0.5125237		0.5003906	
		$\hat{S}$	0.4907468	0.4867546	0.4936833	0.5034568	0.49	0.4846331
		IMSE	0.00749393	0.007419209	0.006488271	**0.005027539**	0.008588674	0.00854753
	1	$\hat{\theta}$	1.062055	1.059707	0.9850451	0.9722513	1.05888	1.059299
		S_{real}	0.4998341		0.500089		0.5000148	
		$\hat{S}$	0.4891757	0.4893821	0.4854448	0.4846246	0.4909491	0.4901453
		IMSE	0.00761582	0.007230105	0.004786318	**0.004225086**	0.008025388	0.007589515
	5	$\hat{\theta}$	5.431239	4.426613	5.28665	3.065479	5.484867	4.422854
		S_{real}	0.4998087		0.4853849		0.5021235	
		$\hat{S}$	0.4890165	0.5424683	0.4840513	0.3960393	0.48846	0.5443319
		IMSE	0.00782172	0.005842755	0.006060699	0.01098205	0.008040944	**0.00576531**
50	0.5	$\hat{\theta}$	0.5047183	0.5057312	0.4942989	0.4976822	0.5065527	0.508078
		S_{real}	0.5000205		0.5061776		0.5007984	
		$\hat{S}$	0.4979477	0.4971327	0.5009317	0.502592	0.4982853	0.4971878
		IMSE	0.00159834	0.001597603	0.001106521	**0.0010536**	0.001653221	0.001653735
	1	$\hat{\theta}$	1.013533	1.01363	0.9747322	0.973215	1.013641	1.014314
		S_{real}	0.499912		0.5024217		0.4995107	
		$\hat{S}$	0.4969847	0.4969322	0.4955623	0.4953384	0.4972758	0.4970205

		IMSE	0.00147171	0.001459014	0.001027533	**0.001007122**	0.00173708	0.001720005
	5	$\hat{\theta}$	5.057885	4.908917	5.024391	4.693546	5.059217	4.904544
		S_{real}	0.4978962		0.5021871		0.4986544	
		$\hat{S}$	0.4969101	0.5054586	0.5011899	0.4889453	0.4976497	0.5064219
		IMSE	0.00151379	0.001453957	0.001089321	**0.001053934**	0.001542883	0.0014815
100	0.5	$\hat{\theta}$	0.5026287	0.5031365	0.5006196	0.5022787	0.5032437	0.504008
		S_{real}	0.5000418		0.4992305		0.5002931	
		$\hat{S}$	0.4988609	0.4984517	0.4983388	0.4991362	0.4990012	0.4984503
		IMSE	0.00083937	0.000839246	0.000546428	**0.000536927**	0.000819200	0.000819491
	1	$\hat{\theta}$	1.007495	1.007571	0.998643	0.99775	1.005334	1.005713
		S_{real}	0.5005186		0.5009175		0.4999311	
		$\hat{S}$	0.4987202	0.4986884	0.4994486	0.4993032	0.4991469	0.4990117
		IMSE	0.00070753	0.000704582	0.000459200	**0.000454114**	0.000778397	0.000774607
	5	$\hat{\theta}$	5.026271	4.953971	4.969366	4.812915	5.035133	4.959745
		S_{real}	0.4989675		0.5013502		0.4995615	
		$\hat{S}$	0.4985826	0.5027586	0.4989795	0.4931558	0.4987254	0.5030291
		IMSE	0.00070227	0.000690020	**0.000662434**	0.000666388	0.000717059	0.000700663

Remarks:

WE note from Tables (1,2,3) following

1- The XG1D distribution is the best among the studied distributions, because it achieved the lowest integral error rate (IMSE) in the simulation experiments.
2- We clearly see that the Bayes estimation method for XG1D was better than the MLE method, although the MLE method was good at estimating.

6. Application

Data were collected from patients infected by Coronavirus at Iraq - Najaf Al-Ashraf - Al Amal Hospital for Infectious Diseases, representing the times of survival (in days) until death or recovery due to Coronavirus infection, in January 2022 with a total of (53) patients, (8,7,1,1,1,5,3,11,10,4,1,5,10,13,7,2,2,8,1,12,5,6,3,3,5,5,8,7,16,9,3,7,47, 15,2,3,15,36,6,37,7,5,2,7,46,1,7,3,30,3,4,4,1) The following results were obtained by analyzing the data using X_c^2 statistics for good fit using the R language:

Table5. Results of the data fit test for the xgamma1 distribution

Dis.	df	X_c^2	X_t^2	α	Decision
Xgamma1	4	1.2453	9.49	0.05	Accept H_0

Remarks:

As can be seen from Table (5), X_c^2 is calculated as (1.2453) and is less than the value of X_t^2 tabular. As a result, the null hypothesis is accepted, meaning the real data are distributed according to xgamma1.

For the purpose of determining which distribution was best when applied to real data, xgamma1 distributions, xgamma distributions, and Lindley distributions were compared. AIC (Akaike information criterion), AICc (consistent Akaike information criteria) and BIC (Bayesian information criterion) were used for model selection; the results are presented in Table (6):

Table 6. Results of the goodness of fit.

Distribution	**AIC**	**AICc**	BIC	HQIC
Xgamma1	338.7622	338.8406	340.7325	14.40236
Xgamma	360.8228	360.9012	362.7931	14.5289
Lindley	348.152	348.2304	350.1222	14.4572

In Table 6, it can be seen that the AIC, AICc, and BIC values of the xgamma1 distribution are smaller than those of the other distributions (xgamma, Lindley), so the new distribution is a very competitive model. As a result, the Xgamma1 distribution fits the data better.

Conclusion

An exponential-gamma mixture, known as Xgamma1, was derived. The distribution was analyzed from various mathematical and structural perspectives. We derived and discussed important survival properties such as the hazard rate. An algorithm for simulation and stochastic ordering was also proposed. Xgamma1 distribution was observed to have added flexibility with regard to certain important properties. We proposed two methods for estimating parameters: maximum likelihood and Bayesian. An application of the Xgamma1 distribution was demonstrated through a simulation study. We also compared the distribution to the xgamma distribution and to the Lindley distribution using a real data set. According to the results, the Xgamma1 distribution is an adequate fit to the data set. Xgamma1 may be used in future applications under different types of censoring strategies.

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