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BAYESIAN RECIPROCAL ADAPTIVE LASSO QUANTILE REGRESSION: SIMULATION AND REAL DATA ANALYSIS

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Abstract

Regression analyses concerns in the explanation and the prediction of the relation between the response variable and a set of predictor variables. The explanation of the regression model obtained via variable selection procedure, while the prediction accuracy of the regression model of the obtained by trading off the bias and the variance of the estimator. This paper discuss the employing the Bayesian adaptive lasso penalized function in quantile regression. The reciprocal adaptive lasso works as variable selection procedure. We employed the scale mixture of normals and the scale mixture of uniforms to develop the Gibbs sampler algorithm. Full conditional posterior distributions have derived based of the hierarchical prior model. Two simulation scenarios and real data analysis conducted to test the performance of the two proposed Bayesian adaptive lasso methods in quantile regression.

Keywords: Reciprocal adaptive lasso, Gibbs sampler algorithm, prior distributions, posterior distributions.

Introduction

Quantile regression analysis one of the most popular regression models in the statistical theory, it is a generalized form of the linear regression model. Also, quantile regression model can be viewed as method that detect more than one relationships (estimated models) between response variable and predictor variables. So, quantile regression model is a robust model. Where the coefficients regression estimates are not affected by the outliers unlike the estimates of the least squares method. Many of data analysts are not interested in the mean regression model estimating of relationship of the response variable and the predictor variables, or sometimes the assumptions of the mean regression violated; such as the normality of the error term, Chatterjee and Hadi (2013). Consequently, if the error term distribution is not specified or the violation of mean regression assumptions, the modeling with other quantities (quantiles) might be more wanted in specifying the exact model. Koenker and Bassett in 1978 introduced the regression quantiles. Marasinghe (2014) stated that the quantile regression estimators are robust and does not require the condition that imposes on the distribution of the error term. In 1987 Koenker and Dorey modified and developed an efficient computing algorithm for estimating the quantile regression parameter estimates. Tibshirani (1996) introduced the lasso method as variable selection procedure with frequents estimation methods. In 1999 Koenker and Machado developed a goodness of fit test for quantile regression model by using the coefficient of determination. In 2001 Yu and Moyeed discussed using of asymmetric Laplace distribution as likelihood function in Bayesian

quantile regression model. In 2006 Zou proposed new penalized function that adds to the residual sum of squares and named adaptive lasso. In 2007 Yu and Stander discussed the Bayesian reference for Tobit quantile regression. In 2008 Li and Zhu studied the variable selection in lasso quantile regression. In 2010 Leng introduced the Bayesian adaptive lasso assuming that tuning parameter takes different values. Kozumi and Kobayashi (2011) proposed new Gibbs sampler algorithm in Bayesian quantile regression assuming tht the asymmetric Laplace distribution can be represents as scale mixture of normal- exponential density. In 2017 Alhusseini introduced the variable selection in Bayesian lasso quantile regression by assuming that scale mixture unifroms. In 2020 Almusaedi and Flaih studied the Bayesian parameter estimation of the quantile regression based on asymmetric Laplace distribution. In 2021 Almusaedi and Flaih Studied the Penalized Bayesian Elastic Net Quantile Regression. In 2020, mallick et al. proposed two reciprocal lasso regression models based on the scale mixture of normal and the scale mixture of uniform. In 2021, Alhamzawi and mallick introduced the reciprocal lasso quantile regression in Bayesian estimation. This paper have new simple and efficient Gibbs sampler algorithm to generate the samples from the target posterior distributions. The simulation results showed that the proposed methods are comparable with other methods.

Bayesian Reciprocal Adaptive Lasso Quantile Regression

Regularization methods are usually used to overcome the problem of ill-conditioned matrix $\boldsymbol{X}$ ($X^T X$ is not inverted) and/or the problem of many predictor variables in the regression model. Lately, high-dimensional shrinkage methods and variable selection procedure have been of great importance. In this section, we proposed a new Gibbs Sampler algorithms based on new hierarchical model for Bayesian reciprocal adaptive lasso quantile regression (BrALqr). This Bayesian model has developed through using the proposed scale mixtures of Mallick et al. (2020) that represents the prior distribution of interested parameters as the inverse Laplace distribution. The inverse Laplace distribution takes the following prior form,

$$\pi(\beta) = \prod_{j=1}^{k} \frac{\lambda}{2\beta_j^2} e^{-\frac{\lambda}{|\beta_j|}} \quad I(\beta_j \neq 0) \cdots\cdots\cdots (1)$$

Mallick et al (2020) proved that the Bayesian reciprocal lasso method is more efficient in computation algorithms that provides efficient convergence in implementing to generates samples from the posterior distribution of the interested parameters, for more details check out Song (2014) and Shine et al. (2018).

$$h(\beta) = \underset{\beta}{\operatorname{armin}}\{\, loss\ function + \sum_{j=1}^{p} \frac{\lambda_j}{|\beta_j|}\} \; I(\beta_j \neq 0)$$

We will employ the Bayesian reciprocal adaptive Lasso quantile regression using scale mixture of uniforms referred to as (BRALQRU) and Bayesian reciprocal adaptive Lasso quantile regression using scale mixture of normals referred to as (BRALQRN), Alhamzawi and Mallick (2020). We

can rewrite the Bayesian minimization problem of the reciprocal lasso quantile regression (2.7) as follows:

$$\min_{\beta} \sum_{i=1}^{n} \rho_{\tau}(y_i - x_i'\beta) + \sum_{j=1}^{p} \frac{\lambda_j}{|\beta_j|} I\{\beta_j \neq 0\}.$$

Hierarchical Priors model of the scale mixture of Uniform

Alhamzawi and Mallick (2020) introduced the parameter estimation in Bayesian reciprocal adaptive Lasso quantile regression by using the scale mixture of uniforms as representation for the prior distribution (1),

$$\frac{\lambda_j}{2\beta_j^2} e^{-\frac{\lambda_j}{|\beta_j|}} = \int_{u_j > \frac{1}{|\beta_j|}} \frac{1}{2u_j\beta_j^2} \frac{\lambda_j^2}{\Gamma(2)} u_j^{2-1} e^{-\lambda_j u_j} du_j, \lambda_j > 0 \quad \dots \quad (2)$$

The hierarchical model of BRALQRU based on (1) and (2) defined as follows

$y_i = x_i^T\beta_\tau + \theta_\tau v_i + \alpha_\tau\sqrt{\sigma v_i} z_i,$

$y_i|x, \beta, \sigma, v \sim \prod_{i=1}^{n} N(x_i^T\beta_\tau + \theta_\tau v_i, \alpha_\tau^2\sigma v_i),$

$$v^{n\times 1} \mid \sigma \sim \prod_{i=1}^{n} Exp(\sigma),$$

$$\beta^{p\times 1} \mid u \sim \prod_{j=1}^{p} \frac{1}{Uniform(-u_j, u_j)}, \quad \dots (3)$$

$$u^{p\times 1} \mid \lambda_j \sim \prod_{j=1}^{p} Gamma(2, \lambda_j),$$

$$\sigma \sim \sigma^{-a-1} \exp\left(-\frac{b}{\sigma}\right),$$

$$\lambda_j \sim {\lambda_j}^{c-1} \exp(-d\lambda_j).$$

Posterior Distributions of BRALQRU

The hierarchical model (3) can be employed with a Gibbs sampler algorithm. Gibbs sampling algorithm is a Markov Chain Monte Carlo (MCMC) tool that draws iteratively samples from the conditional posterior distribution of a specific variable conditioned on all other variables. The

hierarchical model (3) utilized in such a way that we can formulate the full conditional posterior distributions that easy to simulate from.

1- The conditional distribution of y is defined by :

$y_i|x,\beta,\sigma,v \sim \prod_{i=1}^{n} N(x_i^T\beta_\tau + \theta_\tau v_i\ , \alpha_\tau^2\sigma v_i)$.

2- The full conditional posterior distribution of β is defined by:

$$\beta \mid y,X,v,u,\lambda,\sigma \sim N_p\left(\hat{\beta}, 2\sigma(X'V^{-1}X)^{-1}\right)\prod_{j=1}^{p} I\left\{|\beta_j| > \frac{1}{u_j}\right\}.$$

Where $\hat{\beta} = (X'V^{-1}X)^{-1}X'V^{-1}(y-\theta v)$, and $V = diag(v_1,\dots,v_n)$.

3- The full conditional posterior distribution of v_i is defined by:

$$v_i \mid y,X,\beta,\sigma,u,\lambda \sim \text{ GIG}\left(\frac{1}{2}, \frac{(y_i - x_i'\beta)^2}{2\sigma}, \frac{1}{2\sigma}\right)$$

Where, GIG is generalized inverse Gaussian.

4- The full conditional posterior distribution of σ is defined by:

$$\sigma \mid y,X,\beta,v,u,\lambda \sim \text{IG}\left(a + \frac{3n}{2}, b + \frac{1}{2}(y - X\beta - \theta v)'V^{-1}(y - X\beta - \theta v)\right).$$

Where, IG is inverse gamma.

5- The full conditional posterior distribution of u is defined by:

$$u \mid y,X,\beta,v,\lambda,\sigma \sim \prod_{j=1}^{p} \text{Exp}(\lambda)\, I\left\{u_j > \frac{1}{|\beta_j|}\right\}.$$

6- The full conditional posterior distribution of λ is defined by:

$$\lambda \mid y,X,\beta,v,u,\sigma \sim \text{Gamma}\left(c + 2p, d + \sum_{j=1}^{p} \frac{1}{|\beta_j|}\right).$$

Hierarchical Priors model based on the scale mixture of Normal

Mallick et al. (2020), Alhamzawi and Mallick (2020) introduced the following proposition based on the work of Armagan et al. (2013):

$\beta \sim N(0,\gamma)I\{|\beta| > \eta\}, \gamma \sim \exp(\zeta^2/2)$, $\zeta \sim \exp(\eta)$, and $\eta \sim Inverse\ Gamma\ (2,\lambda)$, then β distributed according to inverse Laplace distribution with parameter λ.

From this proposition, the inverse Laplace distribution can be represents as scale mixture of truncated normal. Now based on the above proposition, minimization problem (2), and the quantile regression model , the hierarchical prior model defines as follows:

$y_i|x,\beta,\sigma,v \sim \prod_{i=1}^{n} N(x_i^T\beta_\tau + \theta_\tau v_i\ , \alpha_\tau^2\sigma v_i)$.

$$\beta^{p\times 1} \mid \gamma, u \sim \prod_{j=1}^{p} N\left(0,\gamma_j^2\right) I\left\{\left|\beta_j\right| > \frac{1}{u_j}\right\}$$

$$\gamma^{p\times 1} \mid \zeta \sim \prod_{j=1}^{p} \operatorname{Exp}\left(\zeta_j^2\right)$$

$\zeta^{p\times 1} \mid u \sim \prod_{j=1}^{p} \operatorname{Exp}\left(\frac{1}{u_j}\right)$, Where $u = \frac{1}{\eta}$

$u^{p\times 1} \mid \lambda \sim \prod_{k=1}^{p} \text{Gamma}\ (2,\lambda)$ … (4)

$$\sigma \sim \sigma^{-a-1}\operatorname{exp}\left(-\frac{b}{\sigma}\right)$$

$$\lambda \sim \lambda^{c-1}\exp\ (-d\lambda)$$

BRALQRN sampling computation:

Calculation of MCMC iterations for drawing randomly samples from the full conditional posterior distributions can be done by the following algorithm steps:

1- Sampling y_i: this can be done by drawing samples from truncated normal with mean $x_i^T\beta_\tau + \theta_\tau v_i$ and variance $\alpha_\tau^2\sigma v_i$.

2- Sampling v^{-1}: this can be done by drawing samples from inverse Gaussian:
$v^{-1}\backslash . \sim \prod_{i=1}^{n} \text{Inverse} - \text{Gaussian}\left(\frac{1}{2}, \frac{1}{|y_i - x_i'\beta|}, \frac{1}{2\sigma}\right)$

3- Sampling u_i: this can be done by drawing samples from
$u\backslash . \sim \prod_{k=1}^{p} \text{Exponential}(\lambda) I\left\{u_k > \frac{1}{|\beta_k|}\right\}$

4- Sampling τ^{-1}: this can be done by drawing samples from inverse Gaussian $\tau^{-1}\backslash . \sim \prod_{k=1}^{p} \text{Inverse} - \text{Gaussian}\left(\frac{1}{2}, \sqrt{\frac{\zeta_k^2}{\beta_k^2}}, \zeta_k^2\right)$.

5- Sampling ζ: this can be done by drawing samples from gamma distribution $\zeta\backslash . \sim \prod_{k=1}^{p} \text{Gamma}\left(2, \left(|\beta_k| + \frac{1}{u_k}\right)\right)$.

6- Sampling β: this can be done by drawing samples from truncated multivariate normal distribution:
$N_P((X'\Omega^{-1}X + T^{-1})^{-1}X'\Omega^{-1}(\mathrm{y} - \theta\mathrm{v}), (X'\Omega^{-1}X + T^{-1})^{-1}) \prod_{k=1}^{p} I\left\{|\beta_k| > \frac{1}{u_k}\right\}$.

7- Sampling σ: this can be done by drawing samples from inverse gamma distribution
$$\mathrm{Inverse} - \mathrm{Gamma}\left(\mathrm{a} + \frac{3n}{2}, b, \frac{1}{4}(y - X\beta - \theta v)'V^{-1}(y - X\beta - \theta v)\right)$$

8- Sampling λ: this can be done by drawing samples from gamma distribution
$\mathrm{Gamm\,a}\left(c + 2p, d + \sum_{k=1}^{p} \frac{1}{|\beta_k|}\right)$.

Simulation Study Analysis

simulation studies and real data analysis have conducted to illustrate the performance of the proposed approaches (Bayesian reciprocal adaptive Lasso quantile regression using scale mixture of uniforms referred to as 'BrALqr.U' and Bayesian reciprocal adaptive Lasso quantile regression using scale mixture of normals referred to as 'BrALqr.N'). The proposed approaches are compared with some existing Bayesian (Alhamzawi et al. (2011), Alhamzawi and Ali (2018), Alhamzawi and Ali (2020) and Alhamzawi (2021)) and non-Bayesian approaches. The approaches in this comparison include:

- Bayesian reciprocal adaptive Lasso quantile regression using scale mixture of uniforms (BrALqr.U).
- Bayesian reciprocal adaptive Lasso quantile regression using scale mixture of uniforms (BrALqr.N).
- Bayesian reciprocal Lasso quantile regression using scale mixture of uniforms (BrLqr.N).
- Bayesian Lasso quantile regression (BLqr).
- Bayesian bridge quantile regression (BBqr).
- Lasso regression (lasso).
- Quantile regression (qr).
- Quantile regression with L1 penalty (qrL1).

We consider two simulation studies:

- Simulation study 1 (very sparse case): $\beta = (4,0,0,0,0,0,0,0,0,0)$.
- Simulation study 2 (inconsistent regardless of the sample size (Zou, 2006): $\beta = (5.6,5.6,5.6,0)$.

The data in the simulation examples were generated by
$$y_i = X_i'\beta + e_i, i = 1,2, \dots, n$$

We setup the error distribution e_i so that the q-th quantile equal to 0. Following Li, et. al (2010), we consider four error distributions:

- $N(\mu, 9)$, we setup μ so that the qth quantile equal to zero.
- $0.1N(\mu, 1) + 0.9\ N(\mu, 5)$, , we setup μ so that the qth quantile equal to zero.
- Laplace distribution, Laplace (μ; b = 3), we setup μ so that the qth quantile equal to zero.
- Mixture of two Laplace distribution, 0.1 Laplace (μ; b = 1) + 0:9 Laplace (μ; b =$\sqrt{5}$), we setup μ so that the qth quantile equal to zero. (Li et al. ,2010).

For the first three simulations (Simulation study 1, Simulation study 4), the rows of the design matrix **X** were generated from $N(0, \sum)$,where $\sum$ has an autoregressive correlated matrix, where $\sum_{ij} = 0.5^{|i-j|}$ for all $1 \leq i \leq j \leq p$. The data for Simulation 4 is following the setup of Zou (2006), where the $cor(x_i, x_j) = -0.39$ for i < j < 4 and $cor(x_1, x_4) = 0.23$, i < 4 . In each simulation study, we run 100 replications. For each replication, we simulate 20 observations as a training set and 200 observations as a testing set. We run the Bayesian algorithms for 13000 iterations discarding the first 1000 iteration as a burn-in. Approaches are compared using median of mean absolute deviation (MMAD):

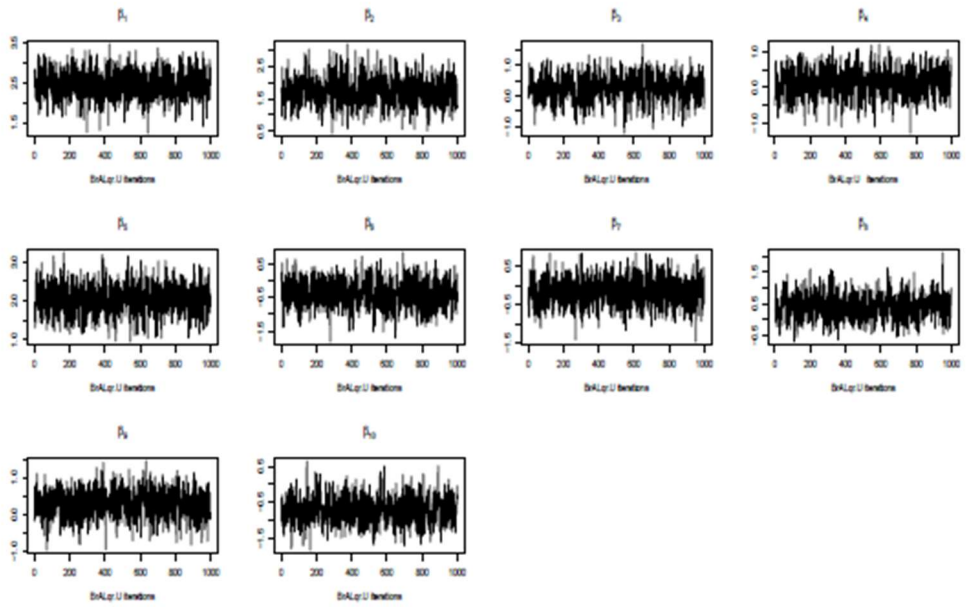
$$\text{MMAE} = \text{Median (mean } \left| x_i^T \beta^{predicted} - x_i^T \beta^{true} \right|)$$

where me is the median which is taken over 100 simulations. The results of the simulations are listed in Tables 1, and 2. We can see that our proposed approaches (BrALqr.U and BrALqr.N) perform well compared with the other existing approaches. For all the simulated cases, convergence of the corresponding MCMC Gibbs sampler was evaluated by trace plots and histograms of the simulated samples. Trace plot is a convergence diagnoses technique, commonly is using to indicate if the generated samples from MCMC for the posterior distribution of parameters convergence to stationary distribution. Moreover, the histograms are used for checking the distribution class of the interested variable.

Table 1: MMADs and SD for Simulation study 1. In the parentheses are standard deviations of the MMADs.

q	Method	Error Distribution			
		normal	normal mixture	Laplace	Laplace mixture
$q = 0.1$	BrALqr.U	2.1493 (0.1373)	1.6336 (0.1738)	2.4785 (0.1942)	2.3083 (0.1751)
	BrALqr.N	2.5076 (0.2328)	1.5859 (0.1857)	2.1649 (0.2701)	2.0547 (0.2024)
	BrLqr.N	2.2091 (0.2527)	1.6231 (0.1003)	2.4169 (0.2196)	1.9508 (0.2106)
	BLqr	**1.2486** (0.3110)	**0.9142** (0.0636)	**1.4334** (0.2444)	1.2821 (0.1364)
	BBqr	2.3263 (0.3175)	1.5358 (0.1085)	2.3227 (0.1486)	2.1511 (0.2002)
	lasso	1.6381 (0.2912)	1.1054 (0.1736)	1.4346 (0.2992)	**1.2706** (0.2393)
	qr	2.5104 (0.2133)	1.5890 (0.2101)	2.1667 (0.2852)	2.0526 (0.1883)
	qrL1	2.1088 (0.4368)	2.1954 (0.3198)	3.2173 (0.3620)	1.7146 (0.1525)
$q = 0.5$	BrALqr.U	**2.0152** (0.2396)	**1.0195** (0.1553)	**1.0796** (0.2912)	1.5004 (0.1380)
	BrALqr.N	2.2917 (0.3010)	1.7263 (0.2562)	2.2254 (0.3663)	1.4353 (0.1340)
	BrLqr.N	2.2325 (0.2252)	1.5421 (0.1075)	2.3257 (0.2334)	1.3516 (0.0963)
	BLqr	2.1878 (0.2388)	1.0861 (0.1256)	1.1580 (0.2942)	1.0115 (0.0839)
	BBqr	2.1616 (0.2329)	1.5721 (0.2130)	2.1773 (0.3042)	1.5150 (0.0773)
	lasso	2.2024 (0.2721)	1.0398 (0.1290)	1.9325 (0.2754)	1.5557 (0.2030)
	qr	2.2902 (0.2999)	1.7309 (0.2601)	2.2267 (0.3482)	1.4787 (0.1257)
	qrL1	2.1348 (0.2001)	1.0947 (0.1362)	1.2447 (0.2865)	**0.9998** (0.2288)
$q = 0.9$	BrALqr.U	2.1182 (0.1700)	1.3553 (0.1288)	**1.0655** (0.2618)	1.0415 (0.2438)
	BrALqr.N	2.5569 (0.2637)	1.3337 (0.1387)	1.1598 (0.3562)	**1.1275** (0.2468)
	BrLqr.N	2.1964 (0.2389)	1.2845 (0.1059)	2.5863 (0.1595)	1.8439 (0.1070)
	BLqr	1.5868 (0.3869)	0.6866 (0.0642)	1.1075 (0.1851)	1.0432 (0.1131)
	BBqr	2.3233 (0.2265)	1.1728 (0.1168)	2.3274 (0.2534)	1.7894 (0.1234)
	lasso	**1.4898** (0.2196)	**0.6259** (0.1708)	1.5689 (0.3660)	1.0531 (0.1186)
	qr	2.5565 (0.2649)	1.3336 (0.1353)	2.5608 (0.3353)	1.9282 (0.2639)
	qrL1	3.0678 (0.2005)	1.8189 (0.2706)	2.8172 (0.4808)	2.6873 (0.4265)

In Table 1, we can see that the proposed method BrALqr.U performs better than the other approaches in 4 out 12 cases.



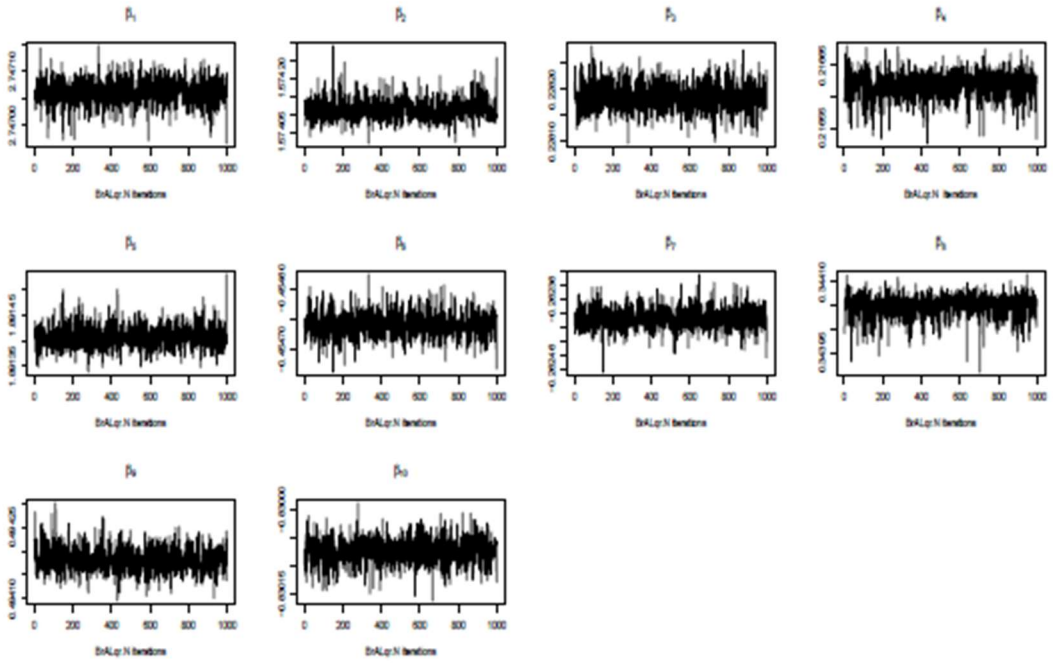
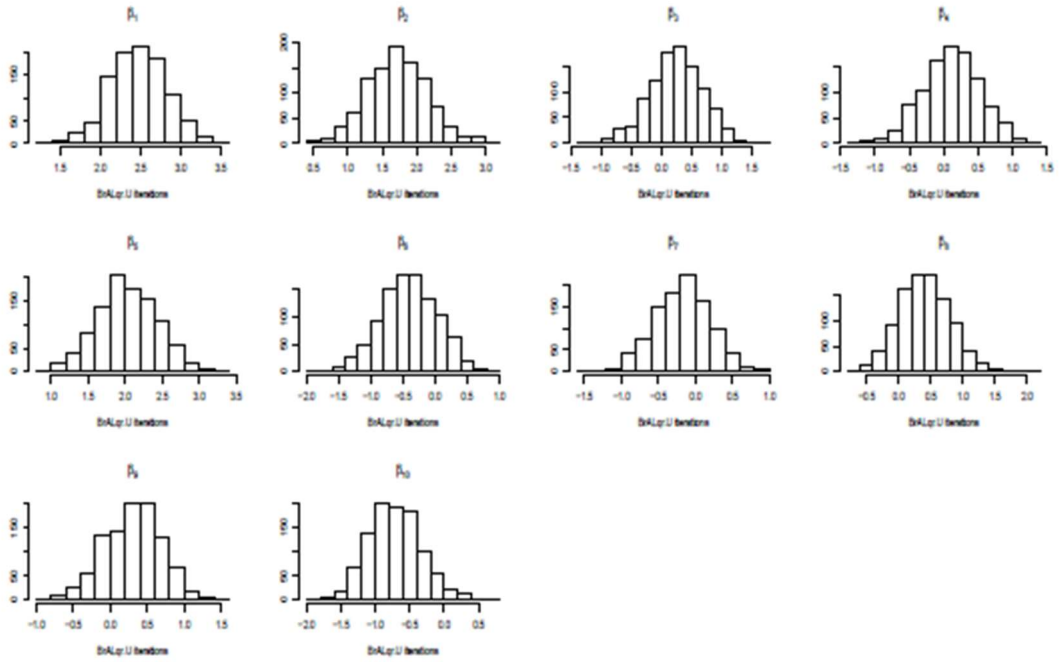


Figure 1: Trace plots based on posterior samples for Simulation 1 when the error is normal and q = 0.9 using BrALqr.U and BrALqr.N methods.

The above figure (1) shows that the trace plots explains no flat bits and that MCMC algorithm suffer no slow mixing which indicates that the proposed methods have good mixing properties. Figure (6) illustrated the distributions of the parameter estimates $\beta_1 - \beta_{10}$ through the histograms and it is clearly that the distribution of the parameters follows the normal distribution.



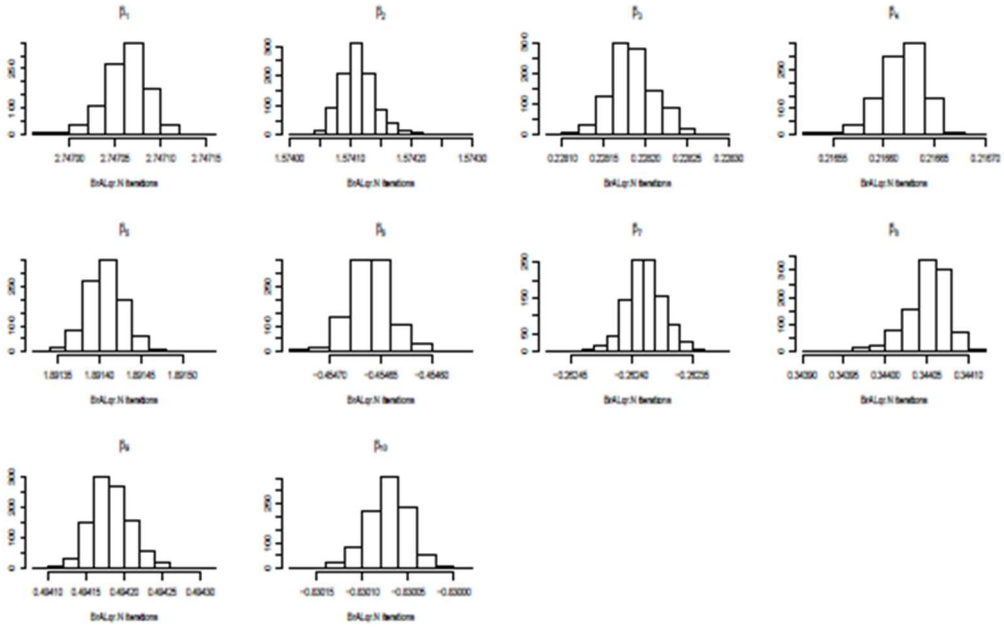


Figure 2: Histograms based on posterior samples for Simulation 1 when the error is normal and q = 0.9 using BrALqr.U and BrALqr.N methods.

Table 2: MMADs and SD for Simulation study 2. In the parentheses are standard deviations of the MMADs.

q	Method	Error Distribution			
		normal	normal mixture	Laplace	Laplace mixture
$q = 0.1$	BrALqr.U	0.6988 (0.0444)	**0.4187** (0.0432)	**0.6245** (0.0762)	0.4895 (0.0312)
	BrALqr.N	0.6182 (0.0444)	0.4459 (0.0306)	0.6688 (0.0654)	**0.4611** (0.0257)
	BrLqr.N	0.7526 (0.0840)	0.5132 (0.0338)	0.6706 (0.0408)	0.5343 (0.0368)
	BLqr	0.7146 (0.0851)	0.5067 (0.0449)	0.7891 (0.0707)	0.5280 (0.0484)
	BBqr	0.7239 (0.0386)	0.4190 (0.0332)	0.6315 (0.0585)	0.5208 (0.0352)
	lasso	**0.5817** (0.0470)	0.4309 (0.0236)	0.7442 (0.0612)	0.5587 (0.0539)
	qr	0.6182 (0.0472)	0.4459 (0.0309)	0.6688 (0.0653)	0.4639 (0.0275)
	qrL	1.9330 (0.0131)	1.9284 (0.0120)	2.0724 (0.0631)	2.0201 (0.0350)
$q = 0.5$	BrALqr.U	**0.6056** (0.0382)	**0.3576** (0.0239)	0.6530 (0.0484)	0.3613 (0.0220)
	BrALqr.N	0.6964 (0.0430)	0.3587 (0.0363)	**0.5711** (0.0266)	**0.3110** (0.0217)
	BrLqr.N	0.7886 (0.0526)	0.3794 (0.0189)	0.6277 (0.0575)	0.3467 (0.0315)
	BLqr	0.6936 (0.0433)	0.3729 (0.0226)	0.6945 (0.0405)	0.3381 (0.0327)
	BBqr	0.7193 (0.0355)	0.4176 (0.0369)	0.5856 (0.0719)	0.3532 (0.0257)
	lasso	0.6179 (0.0503)	0.3636 (0.0212)	0.9203 (0.0920)	0.5282 (0.0363)
	qr	0.6964 (0.0431)	0.3587 (0.0340)	0.5717 (0.0265)	0.3118 (0.0233)
	qrL1	1.9376 (0.0134)	1.9071 (0.0121)	1.9353 (0.0211)	1.9108 (0.0100)
$q = 0.9$	BrALqr.U	0.6786 (0.0532)	0.4339 (0.0260)	0.6525 (0.0663)	0.4638 (0.0433)
	BrALqr.N	0.6319 (0.0389)	**0.4000** (0.0227)	**0.5402** (0.0379)	**0.3908** (0.0304)
	BrLqr.N	0.6110 (0.0353)	0.4343 (0.0340)	0.6988 (0.0410)	0.4740 (0.0299)
	BLqr	0.6552 (0.0662)	0.4414 (0.0251)	0.6508 (0.0615)	0.5147 (0.0373)
	BBqr	0.6891 (0.0586)	0.4448 (0.0210)	0.6806 (0.0467)	0.4860 (0.0367)
	lasso	**0.5287** (0.0286)	0.4262 (0.0396)	0.8162 (0.0709)	0.5908 (0.0319)
	qr	0.6319 (0.0388)	0.4030 (0.0233)	0.5418 (0.0397)	0.3909 (0.0334)
	qrL1	1.9884 (0.0266)	1.9290 (0.0293)	2.1408 (0.0550)	2.0505 (0.0306)

In Table 2, we can see that the proposed method BrALqr.N performs better than the other approaches in 6 out 12 cases while the proposed method BrALqr.U performs better than the other approaches in 4 out 12 cases.

Real Data Analysis

The data was obtained from Al-Kema medical laboratory, which is a laboratory licensed by the Ministry of Health, located in Al-Diwaniyah Governorate. The phenomenon under study is a medical phenomenon in which the dependent variable was (y) and 17 explanatory variables as shown in the table below. The sample size under study was (114) single and after data collection it was dealt with in a standard format to ensure data purification because the units of measurement for the variables are different. We randomly divide the data into a training set with 30 observations and a testing set with 83 observations. Model fitting is carried out on the training set observations and performance is calculated with the mean square error on the testing set, table (3) shows the results which indicates that the proposed method perform better than the other approaches.

Variables description	Abbreviation	Symbol
Seram cholesterol	s. cholesterol	Y
Random Blood sugar	R.B.Sugar	X1
Blood urea	B.Urea	X2
serum creatinine	S.creatinine	X3
low density Lipoproten	LDL	X4
High densite Lipoproten	HDL	X5
Calcium	Ca++	X6
High ceeL Test	HCT	X7
Heamglobin	Hb	X8
Packt cell valume	PCV	X9
Weate Blood cell	WBC	X10
Earthrocet R eate	E.S.R	X11
	Blood group	X12
PLTelate	PLT	X13
Procalci	PCT	X14
mean platelet volume	MPV	X15

	Weight	X16
	Age	X17

Table 3: The case study variables

Table 4: Prediction errors with their standard deviations for the real data (The bold numbers correspond to the smallest MSE in each category.)

	$q = 0.1$	$q = 0.5$	$q = 0.9$
	MSE (SD)	MSE (SD)	MSE (SD)
BrALqr.U	**5007.795 (70.262)**	5173.842 (69.184)	**6062.434 (118.374)**
BrALqr.N	5836.318 (67.070)	**5084.959 (69.102)**	6562.338 (125.629)
BrLqr.N	7931.895 (77.131)	7931.895 (77.131)	32383.217 (128.248)
BLqr	5144.131 (70.424)	5518.731 (71.831)	16829.566 (127.068)
BBqr	7931.852 (77.131)	6123.069 (79.322)	6123.069 (79.322)
lasso	7059.217 (67.662)	7059.217 (67.662)	7059.217 (67.662)
qr	7931.843 (77.131)	6123.071 (79.322)	32383.235 (128.248)
qrL1	10412.950 (68.652)	7537.549 (76.977)	12799.023 (115.022)

Conclusions

The violation of the least squares methods motivates the researchers to search about more valuable parameters estimation methods, the regularization methods that employed the penalized function have been widely popular method in regression analysis, such as lasso, adaptive lasso, elastic net,…etc. The reciprocal adaptive lasso is another regularization method. New hierarchical prior models have introduced with the scale mixture is of normls and the scale mixture of uniforms. Based on the hierarchical prior models the posterior densities have developed, also Gibbs sampler algorithm have implemented for the necessary computations. A comparison between the two proposed models have conducted and to assess the quality of the coefficients estimates through simulation scenarios and real data analysis. The criterion that is named median mean absolute deviation and its standard deviation has used to assess the quality of the parameter estimation methods in simulation results, but the mean square error and its standard error criterion has used to assess the quality of the parameter estimates in real data analysis. Results in both of the simulation scenarios and real data analysis show that the proposed methods are comparable to the other methods.

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