

## ANALYSIS OF MECHANICAL STRUCTURES OF COMPLEX TECHNICAL SYSTEMS

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### Abstract

The work is devoted to the structural analysis of complex technical systems. Mechanical structures are the properties of which affect the behavior of products during assembly, repair and operation are considered.

The main source of data on parts and mechanical connections between them is a hypergraph. This model formalizes a multidimensional basic relation. The hypergraph correctly describes the connectivity and mutual coordination of parts, which is achieved during the assembly of the product. When developing complex products in CAD systems, an engineer often makes serious design mistakes: the relocation of parts and inconsistent assembly operations. Effective ways of detecting these structural defects have been proposed. It is shown that the property of an independent assembly can be represented as a closure operator, the domain of which is the logical value of the set of parts of the product. The images of this operator are connected and coordinated subsets of parts that can be assembled independently. A lattice model is described, which is a space of states of the product during assembly, disassembly and decomposition into assembly units. The lattice model serves as a source of various structural information about the project. Numerical estimates of the power of a set of acceptable alternatives in the problems of choosing the sequence of assembly and decomposition into assembly units are proposed. For many technical operations (control, testing, etc.), it is necessary to mount all parts of the operand in one assembly unit. A

simple formalization of the technical conditions requiring the inclusion (exclusion) of parts in the assembly unit (from the assembly unit) has been developed. A theorem that gives a mathematical description the scheme of decomposition of the product into assembly units in exact terms of the lattice is given. A method of numerical evaluation of the reliability of the mechanical structure of a complex technical system is proposed.

**Keywords:** Mechanical structure, structural analysis, computer-aided design, hypergraph structural model, lattice product model

## Introduction

A mechanical structure (assembly structure, product structure) is a set of parts and components together with mechanical connections (connections and interfaces) that deliver functional integrity and geometric definiteness to a technical system [Whitney, 2004]. The mechanical structure (MS) of a technical system is the most important design decision that affects many technical and economic characteristics of a machine, apparatus, installation or mechanical device. Thus, from it to a large extent the sequence of assembly, decomposition into assembly units, maintainability of the product, etc. depend. In modern computer-aided design systems, there are no special tools for developing and evaluating the quality of MS. This part of the project of a new technical system is formed in many ways automatically, in the process of implementing other design procedures: oblique design, creation of a three-dimensional geometric model, synthesis of dimensional construction schemes, etc. The properties of the mechanical structure are laid down at the beginning of the life cycle, and they are verified at the final stages of technical preparation of production and during the operation of the product [Pirogov S.P.; Ustinov N.N.; Smolin N.I., 2018]. Therefore, an important and urgent task is to develop models and methods of structural analysis of complex products, allowing at the initial stages of the vital cycle of the technical system to assess the quality of the structure and identify structural errors of the project.

The capabilities of automated structural analysis largely depend on the mathematical description of the structural characteristics of the product. Overwhelmingly

The connection graph and its numerous varieties are used as a mathematical model of the mechanical structure: Liaison diagram [Bourjault, 1988], Liaison graph [Karjalainen, 2007], Attributed liaison graph [Vigano, Gomez, 2012], Product liaison graph [Erdos, Kis, Xirouchakis, 2001], Connection graph [De Mello, Sanderson, 1991], Part mating graph [Ko, Lee, 1987], Connective relation graph [Gu, Yan, 1995], etc.

The connection graph is a simple mathematical model that takes into account only mechanical connections between parts. We give an exact formal definition of this model. Denote by  $X = \{x_i\}^n$  the set of parts of the product. A  $i=1$  connection graph is an undirected graph  $G(X, V)$  in which the set of vertices  $X$  corresponds to details, and the set of edges  $V$  corresponds to mechanical connections. Edge  $v = (x, y)$  connects the vertices of  $v \in V$ ;  $x, y \in X$  if and only if between the parts  $x$  and  $y$  have a connection or pairing.

The possibilities of the graph model are small. With its help, it is possible to perform only a shallow structural analysis of the product and calculate the simplest structural characteristics of the project.

For example, to determine the connectivity of a mechanical structure, identify bridges, hanging vertices, cycles and cliques [Bozhko, 2018].

In [De Mello, Sanderson, 1991], a method is described that allows modeling sequential assembly operations by cutting the graph  $G = (X, V)$  into two connected fragments (Bi-partitioning). The set of all permissible cuts is represented in the form of an AND–OR-graph, which can be used to calculate some structural indicators, in particular the product compatibility.

In [Vigan'o, Gomez, 2013], the so-called indices of the centrality of vertices (Centrality index) and the index of the importance of the fundamental cycles of the graph (Importance index) are calculated from the connection graph. These data are used to search for a rational decomposition of the product into assembly units.

In [Bourjault, 1988], one of the first systematic methods of automated design of the assembly sequence of a complex product was proposed. It is based on expert ordering of the edges of the connection graph. An improved version of this method, which requires fewer calls to an expert, is described in [De Fazio, Whitney, 1987].

In [O'shea, Kaebernick, Grewal, 2000], the connection graph is used to decompose a complex product into assembly units. The paper proposes a method that relies only on structural characteristics of the product. We are looking for such a cutting of the graph  $G = (X, V)$  into subgraphs in which the internal connectivity of the parts prevails over the external one.

An original method for analyzing the structural and geometric relationships of a complex product was proposed in [Wilson, Latombe, 1994]. The so-called *db*-graph (Directional blocking graph) serves as a carrier of information about connections. It describes the prohibitions on infinitesimal movements of parts along the selected direction, i.e., local obstacles that create contacting parts. A *Db* graph is a subgraph of a graph  $G = (X, V)$  with oriented ribs. This structural model can be used to analyze geometric obstacles during assembly and disassembly of the product.

The graph of connections  $G = (X, V)$  gives a weak mathematical description of the structural properties of the product, since it takes into account only contacts between parts and components. This model cannot adequately formalize the mutual coordination of parts in the product, since this property, in general, can be achieved by the implementation of several mechanical connections at the same time. In [Bozhko, 2019], a hypergraphic model of the product is described, which is correctly modeled – it demonstrates the connectivity, geometric coordination and assemblability of a technical system (machine or mechanical device). Here are the main definitions and results.

### **Hypergraphic model of mechanical structure**

The hypergraph  $H = (X, R)$  is comparable to the product  $X$ , in which the set of vertices  $X = \{x_i\}^n$  corresponds to parts and components, and the set of hyperdugs

$R = \{r_j\}^m$  describes minimal geometrically defined groupings of parts obtained by basing on internal design bases products. Fig. 1 shows: a fragment of a drum node (a), a graph of connections (b), a hypergraph (c).

In modern engineering practice, the so-called sequencing absolutely prevails alternative coherent assembly operations [Ghandi, Masehian, 2015]. These are operations that implement some

mechanical connection (a set of connections) and are performed with the help of two working bodies that carry out the relative movement of the mounted elements of the structure. Here are some formal definitions necessary for a correct mathematical description of these fundamental properties of assembly operations and processes. Denote  $\{x_1, x_2, \dots, x_{ir}\}$  the set of all vertices incident to the edge  $r \in R$  of the hypergraph  $H = (X, R)$ . The degree of an edge  $r$  is called a number  $|\{x_1, x_2, \dots, x_{ir}\}|$ .

**Definition 1.** By tightening the edge  $r = \{x_1, x_2, \dots, x_{ir}\}$  is called the operation of identifying all vertices  $\{x_1, x_2, \dots, x_{ir}\}$  and exceptions  $r$  from the set  $R$ .

**Definition 2.** Normal contraction is called the contraction of an edge of degree two.

**Definition 3.** A hypergraph  $H = (X, R)$  for which there is a sequence of normal contractions that transforms it into a one-vertex hypergraph without loops (point) is called an  $s$ -hypergraph.

Normal contraction is a mathematical description of coherent and sequential assembly operation.  $S$ -hypergraph — a model of a mechanical structure assembled technical

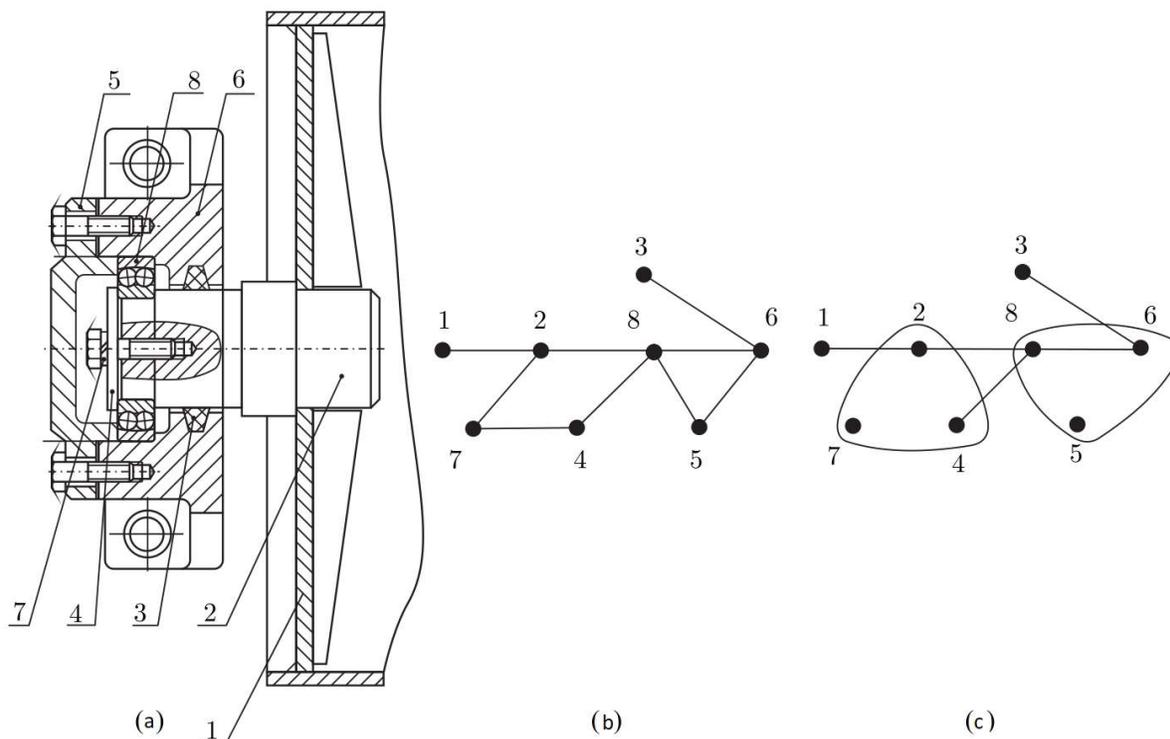


Fig. 1. Fragment of the drum node (a), graph of node connections (b), hypergraph of node (c) of the system, and a single-vertex hypergraph without loops corresponds to an assembled product, all mechanical connections of which are implemented.

**Theorem 1.** Let the hypergraph  $H = (X, R)$  be transformed into a point by sequence of normal contractions. Then:

- 1)  $H = (X, R)$  is connected;
- 2) in the set  $R$  there is at least one edge of the second degree;
- 3) the equality  $|X| = |R| + 1$  is fulfilled.

In [Bozhko, 2012] a formal proof of this important theorem is given.

### Structural analysis based on the hypergraphic model of the product

Here are the technical arguments justifying the equality  $|X| = |R| + 1$ . Let's take some sequence of assembly of the product. The first part in this sequence is installed in the device, so it does not need internal design work. For each subsequent part, exactly one set of internal bases is required. Hence, the validity of the relation  $|X| = |R| + 1$  immediately follows.

#### Rebase

Let the first two conditions of Theorem 1 and  $|X| \ll |R| + 1$  be satisfied for the hypergraph  $H = (X, R)$ . We will call such hypergraphs redundant. It is easy to see that any sequence of normal contractions transforms the redundant hypergraph  $H$  either into a single-vertex graph with loops (Fig. 2) or into an unstretchable graph (Fig. 3).

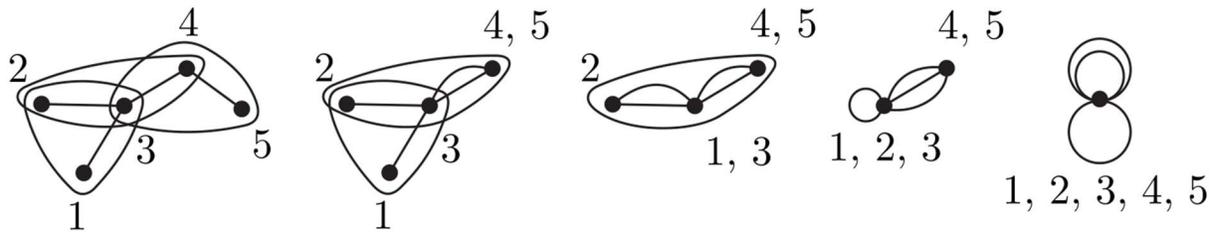


Fig. 2. A sequence of normal contractions that transforms a redundant hypergraph into a point with loops

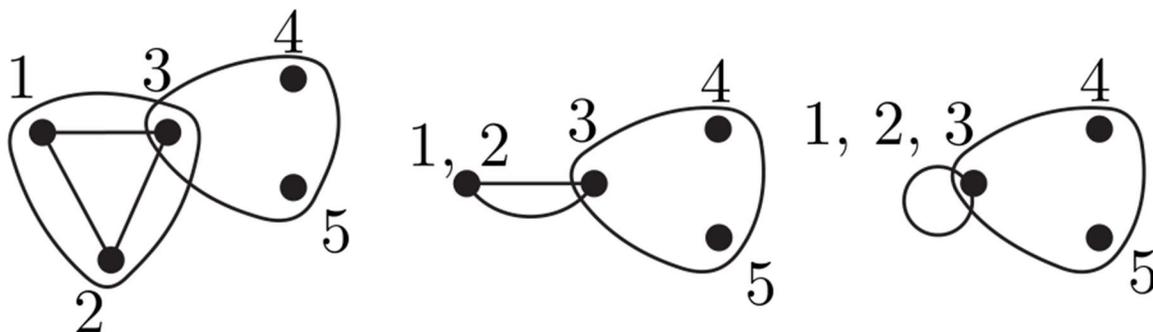


Fig. 3. A sequence of normal contractions that transforms a redundant hypergraph into a non-stretchable hypergraph of the third order

The appearance of loops after the normal contraction operation contradicts the interpretation of one vertex graph (subgraph) as an image of an assembled product (a fragment of a product) in which all connections are implemented. Loops mean the presence of excessive coordination of parts. This is a serious design error, which at the design stage generates unsolvable dimensional

problems, and at the production stage entails relocation. Overbasing is the installation of a part with simultaneous coordination across several complete design bases [Whitney, 2004].

Figure 4, *a* shows a simple bearing mounting design. In this figure: 1 — cup, 2 — bearing, 3 — sleeve, 4 — cover, 5 — gear. Figure 4, *b* shows a hypergraph of this construction.

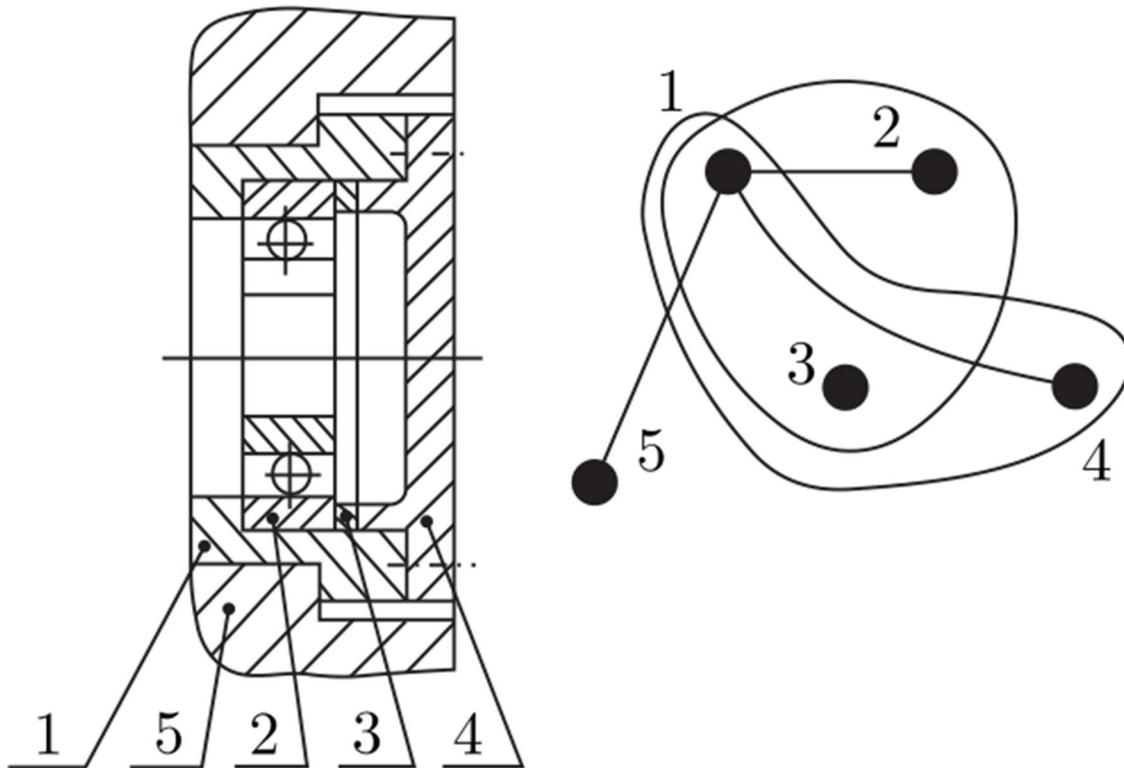


Fig. 4. Bearing mounting structure with relocation (a), redundant hypergraph describing the mechanical structure of this structure (b)

According to the operating conditions, the cover must keep the bushing and the bearing from moving horizontally, so there can be no gap between parts 3 and 4. According to the drawing, it is necessary to implement a connection between the lid 4 and the glass 1. Hence, part 4 must be installed with coordination along two complete bases that form parts 1 and 1, 3. In Fig. 4, *b*, this situation is described by two hyperedges  $\{1, 4\}$  and  $\{1, 3, 4\}$ . This means that the cover is 4 it is mounted with rebasing, due to which an unsolvable dimensional chain arises horizontally. This chain cannot be implemented without non-design deformations of the structure. To remove the rebasing, it is necessary to create a gap between parts 1 and 4, therefore, an edge  $\{1, 4\}$  must be removed in the hypergraph.

***Lack of coordination and lack of sequencing***

Any normal contraction operation preserves the connectivity of the hypergraph and reduces the number of vertices and the number of edges by one. Let conditions 1 and 2 of Theorem 1  $|X| > |R| + 1$  be satisfied for the hypergraph  $H = (X, R)$  describing the mechanical structure of the

product+1. Let's perform all possible normal contractions in  $H = (X, R)$ . As a result, we get a connected hypergraph  $Hi = (Xi, Ri)$ , in which there are no edges of the second degree and  $|Xi| > |Ri| + 1$ ;  $|Xi| - |Ri| + 1 = |X| - |R| + 1$ . Obviously, this hypergraph does not allow for further contraction. Non-extensible hypergraphs of the form  $Hi = (Xi, Ri)$  describe nonsequential or uncoordinated structural fragments of a mechanical system. Uncoordinated are such elements of the structure, which, due to design errors, received “illegal” degrees of freedom.

A few simple examples are shown in Fig. 5. Figure 5 a, a shows a structure that requires three working bodies for assembly (non-sequential). Figures 5, b and 5, c show examples of two simple structures with insufficient coordination in the horizontal direction.

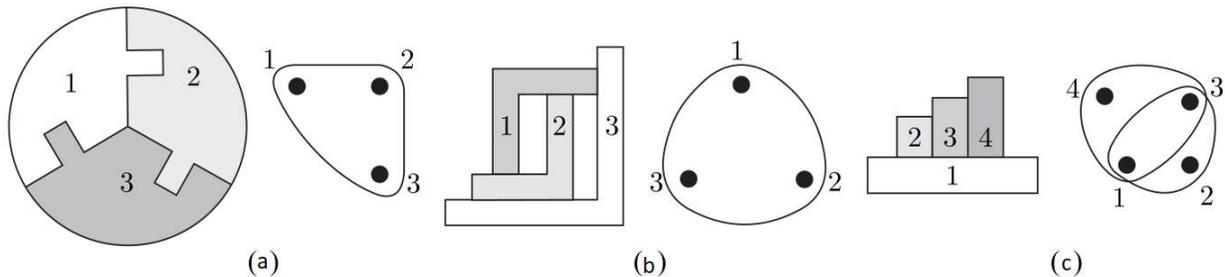


Fig. 5. A non-sequential structure requiring three working bodies for assembly (a); examples of non-ordinated structures with a degree of mobility in the horizontal direction (b, c)

Installation of all three structures is carried out by means of movements on the plane. It is easy to see that the assembly of any two elements of the “puzzle” (Fig. 5, a) prohibits the installation of the third. Therefore, this construction can be obtained only by simultaneous movement of all three elements. To implement these movements, three working bodies are required, i.e., this assembly operation is non-sequential. The examples given do not exhaust the possibilities of structural analysis of complex technical systems provided by the hypergraph model. Let 's list some of the among them: connectivity analysis, search for bridges and overloaded vertices, express assessment of the assemblability and dismemberability of the product, calculation of the upper limit of the lengths of design chains, identification of fragments in violation of the “principle of design proximity”, etc [Bozhko, 2012].

## Structural analysis of the lattice model of the product

### Basic provisions

Let some  $s$ -hypergraph  $H = (X, R)$  be given. Consider the family of all its  $s$ -subgraphs together with the empty set. Let's order this family by including vertices:  $H_a = (X_a, R_a) \leq H_b = (X_b, R_b) \Leftrightarrow X_a \subseteq X_b \quad \forall H_a, H_b \subseteq H$ . Let 's denote this ordered set as  $(F(H), \leq)$ . For what follows, we will need several definitions and results from the solution theory [Roman, 2008].

Let  $X$  be some set and  $B(X)$  be its Boolean.

**Definition 4.** The mapping  $\alpha: B(X) \rightarrow B(X)$  is called a closure operator if

$\forall Y \in B(X)$  the conditions are met:

- 1)  $Y \leq \alpha(Y)$  (extensiveness);
- 2)  $Y_1 \leq Y_2 \Rightarrow \alpha(Y_1) \leq \alpha(Y_2)$  (monotony);
- 3)  $\alpha(\alpha(Y)) = \alpha(Y)$  (idempotence).

**Definition 5.** Element  $Y \in B(X)$  is called closed if  $\alpha(Y) = Y$ .

**Lemma ([Roman, 2008]).** *The set of all  $\alpha$ -closed elements of the Boolean  $B(X)$  is a lattice.*

If an  $s$ -hypergraph  $H = (X, R)$  is given, then any of its generated subgraphs  $Hd = (Xd, Rd), Hd \subseteq H$  is completely determined by the set of vertices  $Xd$ .

**Definition 6.** The set of vertices  $Xd$  of the  $s$ -subgraph  $Hd = (Xd, Rd)$  of the  $s$ -hypergraph  $H = (X, R)$  is called the  $s$ -set.

We show that  $s$ -sets are closed elements with respect to some closure operator acting on the Boolean  $B(X)$ , in which  $X = \{x_i\}^n$  is the set of parts of the product.

Let's define on  $B(X)$  the mapping  $\varphi: B(X) \rightarrow B(X)$ , which corresponds to each subset of the children  $Y \subseteq X$  with  $\varphi(Y) \subseteq X$  the minimum  $s$ -set that includes  $Y$ .

Let's check the validity of properties 1-3 of definition 4.

By definition, the extensivity property holds for  $\varphi$ , i.e.,  $Y \leq \varphi(Y)$ . Let's check the monotony. If for subsets of parts  $Ya, Yb \subseteq X$  there is  $Ya \subseteq Yb$  and, then the minimum  $s$ -set  $\varphi(Yb)$ , which includes  $Yb$ , is an  $s$ -set (and possibly not a minimal one) for  $Ya$ . From here,  $\varphi(Ya) \leq \varphi(Yb)$  immediately follows. Let's justify idempotence. For any  $Y \subseteq X$ , its image  $\varphi(Y)$  is the minimal  $s$ -set, so  $\varphi(\varphi(Y)) = \varphi(Y)$ .

**Theorem 2.** *An ordered set  $(F(H), \leq)$  is a lattice  $(F(H), \wedge, \vee)$ .*

Let's list the main properties  $(F(H), \wedge, \vee)$ . This is a finite atomic lattice, in which:

elements describe connected, coordinated and assembled parts of the product ( $s$ -sets), atoms represent details, the largest element (unit of the lattice) is the product, the named element (zero of the lattice) is an empty set. For any  $s$ -sets  $A, B \in F(H)$ , their lattice intersection  $A \wedge B$  is the maximal  $s$ -set i.e. included in  $A$  and  $B$ . The lattice union  $A \vee B$  is a minimal  $s$ -set including  $A$  and  $B$ . Figure 6 shows the lattice  $F(H)$  of the drum assembly shown in Figure 1.

Connectivity, mutual coordination and independent assemblability are the most important properties of the product and its components. They are necessary for a variety of design and production operations, for example: design, technological preparation of production, assembly, disassembly, repair, control, measurements, trial operation, etc. Therefore, the grid  $F(H)$  allows you to perform a deep and multidimensional analysis of the quality of MS at various stages and operations of the product life cycle.

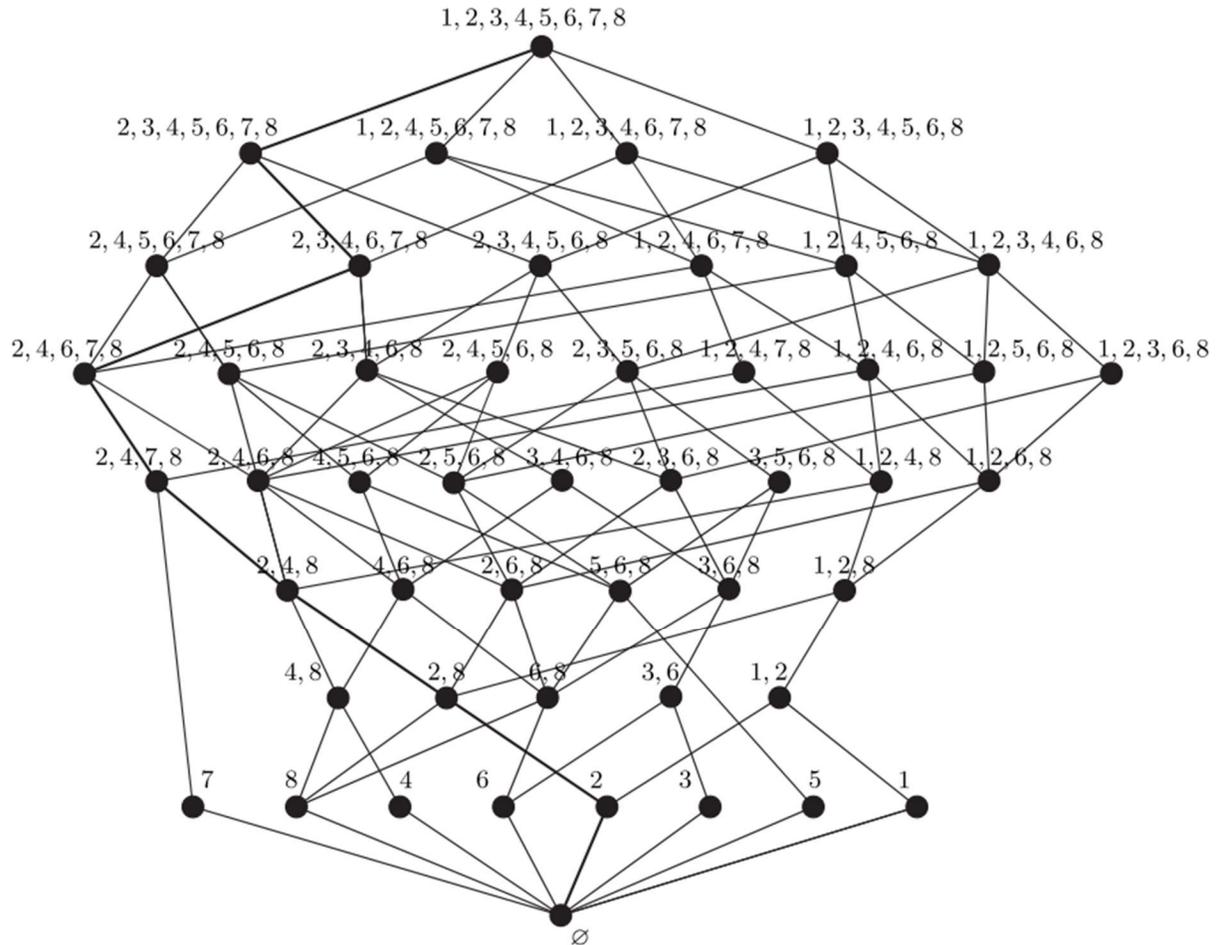


Fig. 6. Lattice model  $(F(H), \wedge, \vee)$  of the drum node shown in Fig. 1, *a*

### Collection analysis

Here are some necessary definitions from the theory of ordered sets and brushes [Roman, 2008].

**Definition 7.** A lattice chain is a nonempty subset of it in which any two elements  $x$  and  $y$  are comparable, i.e.,  $x \leq y$  or  $x \geq y$ .

Chains are usually written according to the ordering of its elements. For example,  $a_1 \leq a_2 \leq \dots \leq a_n$  or abbreviated without order symbols,  $(a_1, a_2, \dots, a_n)$ .

**Definition 8.** It is said that the element  $z$  covers (covers) the element  $x$  if  $x \leq z$  and there is no such  $y$  that  $x \leq y \leq z$ .

**Definition 9.** The maximal chain is called the chain  $a_1 \leq a_2 \leq \dots \leq a_n$  in which each element  $a_{i+1}$  covers  $a_i$ ,  $i = 1, n - 1$ .

The large number of assembly sequences allowed by the mechanical structure of the technical system is an obvious design advantage, which at subsequent stages of the life cycle provides greater freedom of choice to the decision-maker [Surinskiy, D.O., Savchuk, I.V., Solomin, E.V.,

Kovalyov, A.A., 2019]. Conversely, if a design has a limited number of acceptable assembly plans, then an engineer or technologist may face difficulties in choosing a rational alternative.

An accurate estimate of the number of valid assembly sequences is the total number  $(0, 1)$ -chains in the lattice  $F(H)$ . We can offer several approximate (upper and lower) estimates of this characteristic, for example: the width of the lattice (the power of the maximum antichain), the sum of the degrees of the vertices (the number of edges), the sum of the degrees of the totality of important vertices, etc.

The product design creates a system of preferences  $P$  (Precedence relations, Precedence constraints [Ben-Arieh, Kramer, 1994]) on a variety of its parts, which formalizes the engineering experience of assembling technical systems for various purposes. For instance, massive parts with low accuracy should be installed at the beginning, high-precision and easily deformable parts – at the end of the assembly process. Parts included in a large number of dimensional chains should be mounted as soon as possible. In [Wolter, Chakrabarty, Tsao, 1992] it is shown that the structure-Preference tour  $P$  can be represented as a partially ordered set  $(X, \leq_P)$ . I.e.,  $\forall x_a, x_b \in X \ x_a < x_b$ , if for any constructive, technological or other reasons, the  $x_a$  must be installed before the  $x_b$  parts.

The set of all linear continuations  $C(X, \leq_P)$  of partial order  $(X, \leq_P)$  is a collection of assembly sequences generated by the structure of preferences  $P$ . We denote by  $C(F(H))$  the set of  $(0, 1)$ -lattice chains  $F(H)$ . The power of the multiple  $C(F(H)) \cap C(X, \leq_P)$  can serve as a numerical indicator of the consistency of the mechanical structure and the structure of preferences  $P$ . The higher this number, the more freedom of decision-making the engineer may have.

### ***Separability analysis***

The division of a product into assembly units (decomposition) is the most important design decision, on which the organization and economic indicators of discrete production largely depend. The inclusion of parts in one assembly unit (CE) is required for many different technical operations, for example: pilot assembly, fitting, testing, measurements, etc. In all these situations, it is advisable to localize the object of production as much as possible. Ideally, it should be assembled only from the elements that are needed to perform a technical operation. The lattice  $F$  allows us to formalize and evaluate the feasibility of the localization principle.

**Definition 10.** The height  $h(A)$  of an element  $A$  of a finite lattice  $L$  is the length of the longest chain connecting  $A$  to the smallest element in  $L$  [Roman, 2008].

Let the parts  $x, y$  be included in one assembly unit for some reason. By definition of the lattice  $F$ , the image of such a CE is the element of the lattice  $A = x \vee y$ . The closer the element  $A = x \vee y$  is to its arguments in the lattice, the fewer parts are needed to assemble  $A$ , the stricter the localization principle is fulfilled.

Let, for instance, for the drum assembly (Fig. 1), it is required to check the alignment of the cover 5 and the shaft 2. Then, using the lattice  $F$  (Fig. 6), we find the minimum assembled set containing these elements  $\{2, 5, 6, 8\} = \{2\} \vee \{5\}$ . For this example, the principle of locality is not strictly fulfilled, since  $h\{2, 5, 6, 8\} = 4$ ,  $h\{2\} = x\{5\} = 1$ .

Viewing an extended projection when, according to some structural and technological foundations, parts  $x$  and  $y$  should not be included in any intermediate three. As long as there is a minimum three that combines  $x$  and  $y$ , the product itself appears. This requirement can be formed using a simple equality  $x \vee y = X$ .

Here are three important definitions [Roman, 2008].

**Definition 11.** An antichain is a nonempty subset of lattice elements in which any two distinct elements  $x$  and  $y$  are incomparable. I.e., the relations  $x \leq y$  or  $x \geq y$  are not fulfilled for them.

**Definition 12.** A nonempty subset of lattice elements is called upward directed if any two of its elements have an upper bound.

**Definition 13.** A nonempty subset of lattice elements that does not contain zero is called orthogonal if the intersection of any two of its elements is equal to the zero of the lattice. In [Bozhko, 2018] an important theorem is proved, which gives a mathematical description of the assemblies- decompositions in exact lattice terms.

**Theorem 3.** *Hierarchically ordered subset  $(M, \leq)$  of the lattice  $(F(H), \wedge, \vee)$ ,*

$M \subseteq F$  is a mathematical description of the decomposition if:

- 1)  $(M, \leq)$  is an upward—directed ordered set;
- 2) all the atoms of the lattice  $F$  are included in  $(M, \leq)$ ;
- 3) any antichain of the set  $(M, \leq)$  is an orthogonal system in the lattice  $F$ .

This theorem makes it possible to evaluate the ability of MS to generate decompositions suitable for implementation in various production conditions, i.e., a quality of structure that can be called structural flexibility.

### ***Structural heuristics***

In modern integrated CAD/CAM/CAE systems, the mechanical structure of the product is formed at the very beginning of the design process, immediately after the synthesis of a three-dimensional geometric model. At this stage, there is no reliable information about the properties of the production system and the operating conditions of the product, which makes it impossible to accurately and objectively assess the technical properties of the design of the machine or mechanical device. Therefore, it is advisable to evaluate the quality of a mechanical structure by the “freedom of choice” that it provides the decision maker at subsequent stages of the life cycle. The lattice model makes it possible to evaluate the reliability and adaptability of the MS of a complex technical system.

Let the mechanical structures of the two products be represented as  $s$ -hypergraphs in Fig. 7.

Figure 8 shows the lattices  $F(Hc)$  and  $F(Ht)$  generated by the hypergraphs  $Hc$  and  $Ht$ .

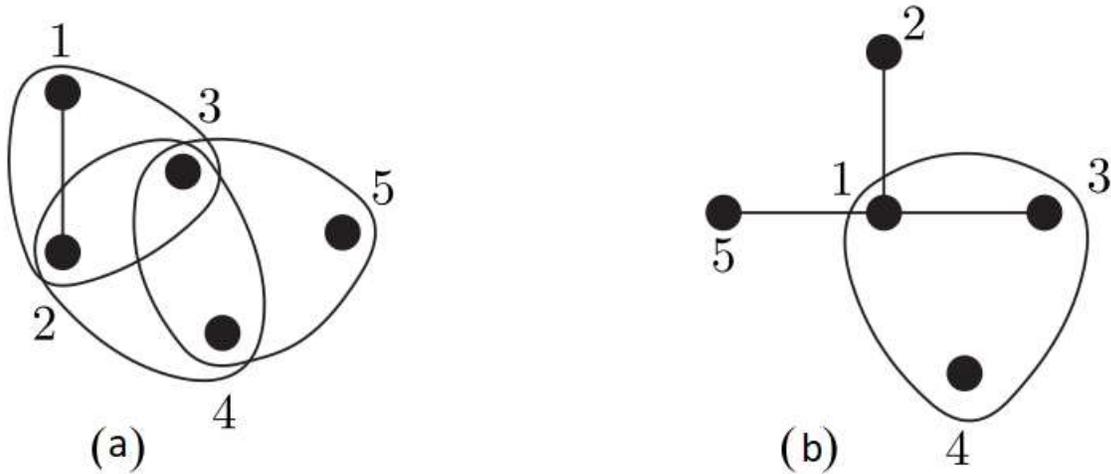


Fig. 7. Examples of fifth-order  $s$ -hypergraphs with different structural properties;  
 $H_c$  (a) и  $H_t$  (b)

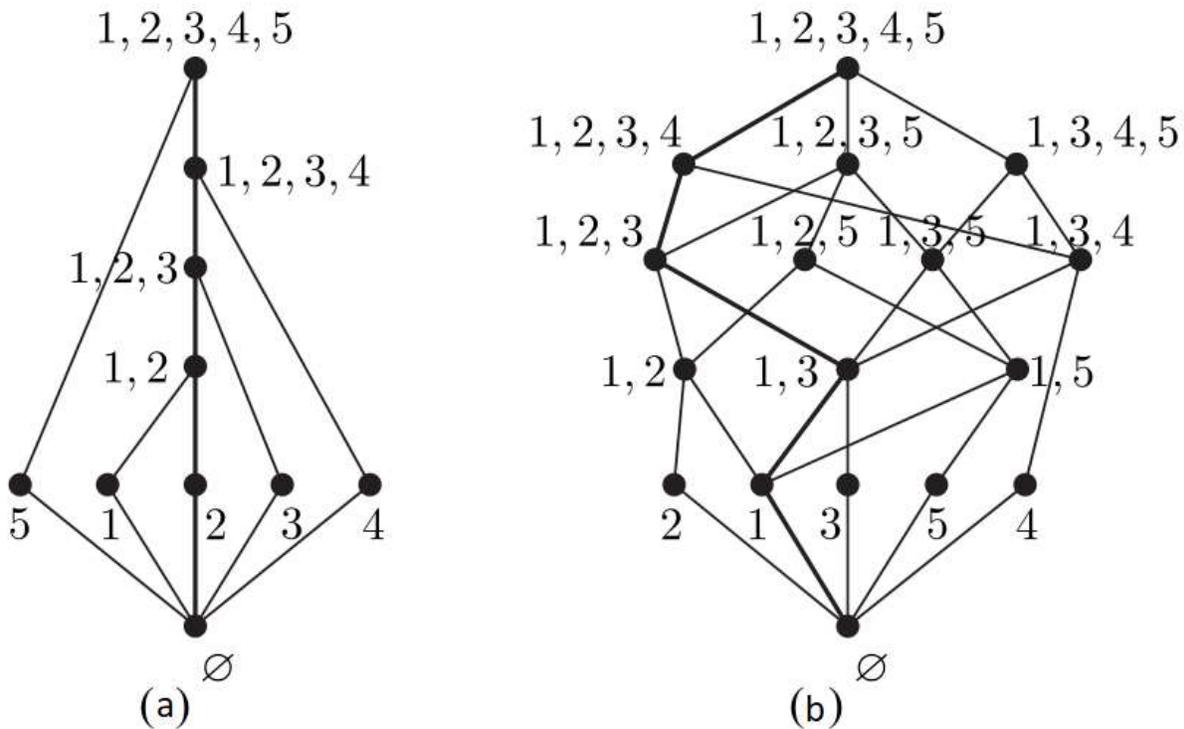


Fig. 8. Lattices  $F(H_c)$  (a) and  $F(H_t)$  (b)

In the lattice  $F(H_c)$  there is only one maximal  $(0, 1)$ -chain (highlighted in bold), which connects the smallest element with the largest, is the chain  $(2, 1, 3, 4, 5)$ . It represents a single linear sequence of contractions (assemblies) of the  $s$ -hypergraph  $H_c$ . It is rigidly predetermined by the mechanical structure and has no alternatives. In this sequence, numerous engineering heuristics

can be violated, which are used to synthesize rational assembly plans, for example: ordering of parts by weight, dimensions, etc. This may be the source of a collision i.e. resolved only by changing the project.

Consider the lattice  $F(Ht)$  (Fig. 8, b), in which the maximum (0, 1)-chain (1, 3, 2, 4, 5). Unlike the example considered, this assembly plan is highly flexible. At each operation, except for the last one, it has several alternative continuations, which make it possible to remove possible technical contradictions.

The lattice model allows us to give a numerical estimate of the robustness of the mechanical structure and the sequence of assembly of the product.

## Conclusion

The mechanical structure is formed at the initial stages of the product life cycle (LC) and has a significant impact on the final stages of LC: assembly production, operation and repair. Structural analysis allows you to quickly identify many design blunders and vulnerabilities of the technical system.

In modern design practice, the most common structural model is the relationship graph. It is shown that this model has limited adequacy. A hypergraphic model of the mechanical structure is proposed, which correctly describes connectivity and geo-metric coordination of parts in the product, achieved with the help of internal design bases. The hypergraph model allows you to perform a deep structural analysis of the project and identify hidden errors at the early stages of design. Methods of identification of relocation, insufficient coordination of details and search for non-sequential structural fragments have been developed.

The development of the hypergraph model is the lattice model of the product. It contains information about all independently assembled parts of the product. This makes it possible to get express assessment of the new design according to the criteria of assemblability, disassemblability and separability into assembly units.

## References

1. Ben-Arieh D., Kramer B. (1994), Computer-aided process planning for assembly: generation of assembly operations sequence. *The international journal of production research*, Vol. 32, No. 3, pp. 643-656.
2. Bourjault A. (1988), Methodology of Assembly Automation: A New Approach. *Robotics and Factories of the Future '87*, pp. 37-45.
3. Bozhko A. (2018), Theoretic-lattice approach to computer aided generation of assembly units. *2018 International Russian Automation Conference (RusAutoCon)*, IEEE, pp. 1-5.
4. Bozhko A.N. (2012). Modeling of positional connections in mechanical systems // *Information technologies*, No. 10, pp. 27-33.
5. Bozhko A.N. (2018), Methods of structural analysis of complex products in integrated CAD/CAM systems. *Information technologies*, Vol. 24, No. 8, pp. 499-506.

6. Bozhko A.N. (2019), Hypergraphic model for the assembly sequence problem. *IOP Conference Series: Materials Science and engineering*. IOP Publishing, Vol. 560, No. 1, p. 012010.
7. De Fazio T., Whitney D. (1987), Simplified generation of all mechanical assembly sequences. *IEEE Journal on Robotics and Automation*, Vol. 3, No. 6, pp. 640-658.
8. De Mello L.S.H., Sanderson A.C. (1991), Representations of mechanical assembly sequences. *IEEE transactions on Robotics and Automation*, Vol. 7, no. 2, pp. 211–227.
9. De Mello L.S.H., Sanderson A.S. (1991), A basic algorithm for the generation of mechanical assembly sequences. *Computer-Aided Mechanical Assembly Planning*, Springer, Boston, Massachusetts, pp. 163-190.
10. Erdos G., Kis T., Xirouchakis P. (2001), Modelling and evaluating product end-of-life options. *International Journal of Production Research*, Vol. 39, no. 6, pp. 1203-1220.
11. Ghandi S., Masehian E. (2015), Review and taxonomies of assembly and disassembly path planning problems and approaches. *Computer-Aided Design*, Vol. 67, pp. 58–86.
12. Gu P., Yan X. (1995), CAD-directed automatic assembly sequence planning. *International Journal of Production Research*, Vol. 33, no. 11, pp. 3069-3100.
13. Karjalainen I. et al. (2007), Assembly sequence planning of automobile body components based on assembly automation.
14. Ko H., Lee K. (1987), Automatic assembling procedure generation from mating conditions. *Computer-Aided Design*, Vol. 19, no. 1, pp. 3-10.
15. O'shea B., Kaebernick H., Grewal S.S. (2000), Using a cluster graph representation of products for application in the disassembly planning process. *Concurrent Engineering*, Vol. 8, no. 3, pp. 158-170.
16. Pirogov S.P.; Ustinov N.N.; Smolin N.I. (2018), Mathematical Model of Stress-Strain State of Curved Tube of Non-Circular Cross-Section with Account of Technological Wall Thickness Variation. *IOP Conf. Ser.: Mater. Sci. Eng.* 357. URL: <https://iopscience.iop.org/article/10.1088/1757-899X/357/1/012037>
17. Roman S. (2008), Lattices and ordered sets. Springer Science & Business Media, 307 p.
18. Surinskiy, D.O., Savchuk, I.V., Solomin, E.V., Kovalyov, A.A. (2019). PV-based energy-saving electro-optical converter development. *19th International Scientific Geoconference SGEM 2019. Conference proceedings*. Vol. 19, No. 4.1, pp. 427-434.
19. Vigano R., Gomez G.O. (2012), Assembly planning with automated retrieval of assembly sequences from CAD model information. *Assembly Automation*.
20. Vigano R., Gomez G.O. (2013), Automatic assembly sequence exploration without precedence definition. *International Journal on Interactive Design and Manufacturing (IJIDeM)*, Vol. 7, no. 2, p. 7989.
21. Whitney D.E. (2004), Mechanical Assemblies: Their Design, Manufacture, and Role in Product Development. New York: Oxford University Press, 518 p.
22. Wilson R.H., Latombe J.C. (1994), Geometric reasoning about mechanical assembly. *Artificial Intelligence*, Vol. 71, no. 2, pp. 371-396.

23. *Wolter J., Chakrabarty S., Tsao J. (1992), Mating constraint languages for assembly sequence planning. Robotics and Automation. Proceedings 1992 IEEE International Conference, Vol. 3, pp. 2367-2374.*